HOOFDSTUK 4: Algebra en meetkunde.

4.1 Rekenen met wortels

Opgave 1:

II, III, IV, VI, VII, VIII

Toelichting:

IV: $\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{\dfrac{18}{2}} = \sqrt{9} = 3$

VI: $\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

VII: $\dfrac{2}{\sqrt{5}} = \dfrac{2}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5} = \tfrac{2}{5}\sqrt{5}$

VIII: $\sqrt{4\tfrac{1}{2}} = \sqrt{2\tfrac{1}{4} \cdot 2} = \sqrt{2\tfrac{1}{4}} \cdot \sqrt{2} = 1\tfrac{1}{2}\sqrt{2}$

Opgave 2:

a. $2\sqrt{3} \cdot 3\sqrt{5} = 6\sqrt{15}$

b. $\dfrac{5\sqrt{10}}{\sqrt{5}} = 5\sqrt{2}$

c. $3a\sqrt{2} \cdot a\sqrt{7} = 3a^2\sqrt{14}$

d. $\dfrac{2\sqrt{14}}{3\sqrt{7}} = \tfrac{2}{3}\sqrt{2}$

e. $\tfrac{1}{2}a\sqrt{2} \cdot \tfrac{1}{2}a\sqrt{3} = \tfrac{1}{4}a^2\sqrt{6}$

f. $\dfrac{6}{5\sqrt{2}} = \dfrac{6}{5\sqrt{2}} \cdot \dfrac{5\sqrt{2}}{5\sqrt{2}} = \dfrac{30\sqrt{2}}{50} = \tfrac{3}{5}\sqrt{2}$

Opgave 3:

a. $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} = \tfrac{1}{3}\sqrt{3}$

b. $\sqrt{\tfrac{1}{2}} = \dfrac{\sqrt{1}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} = \tfrac{1}{2}\sqrt{2}$

c. $\sqrt{4\tfrac{1}{2}} = \sqrt{2\tfrac{1}{4} \cdot 2} = \sqrt{2\tfrac{1}{4}} \cdot \sqrt{2} = 1\tfrac{1}{2}\sqrt{2}$

d. $\left(\tfrac{1}{2}\sqrt{5}\right)^2 = \tfrac{1}{2}\sqrt{5} \cdot \tfrac{1}{2}\sqrt{5} = \tfrac{1}{4} \cdot 5 = 1\tfrac{1}{4}$

e. $\left(\tfrac{1}{2}a\sqrt{2}\right)^2 = \tfrac{1}{2}a\sqrt{2} \cdot \tfrac{1}{2}a\sqrt{2} = \tfrac{1}{4}a^2 \cdot 2 = \tfrac{1}{2}a^2$

f. $\left(\tfrac{2}{3}a\sqrt{3}\right)^2 = \tfrac{2}{3}a\sqrt{3} \cdot \tfrac{2}{3}a\sqrt{3} = \tfrac{4}{9}a^2 \cdot 3 = 1\tfrac{1}{3}a^2$

Opgave 4:

a. $\sqrt{24} + \sqrt{6} = \sqrt{4 \cdot 6} + \sqrt{6} = 2\sqrt{6} + \sqrt{6} = 3\sqrt{6}$

b. $\sqrt{80} - \dfrac{10}{\sqrt{5}} = \sqrt{16 \cdot 5} - \dfrac{10}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = 4\sqrt{5} - \dfrac{10\sqrt{5}}{5} = 4\sqrt{5} - 2\sqrt{5} = 2\sqrt{5}$

c. $\sqrt{18a} - \sqrt{8a} = \sqrt{9 \cdot 2a} - \sqrt{4 \cdot 2a} = 3\sqrt{2a} - 2\sqrt{2a} = \sqrt{2a}$

d. $\sqrt{3a^2} + \sqrt{12a^2} = |a| \cdot \sqrt{3} + \sqrt{4a^2 \cdot 3} = |a| \cdot \sqrt{3} + 2 \cdot |a| \cdot \sqrt{3} = 3 \cdot |a| \cdot \sqrt{3}$

e. $\sqrt{\frac{3}{4} a^2} = \sqrt{\frac{1}{4} a^2 \cdot 3} = \frac{1}{2} \cdot |a| \cdot \sqrt{3}$

f. $\sqrt{\frac{7}{9} a^2} = \sqrt{\frac{1}{9} a^2 \cdot 7} = \frac{1}{3} \cdot |a| \cdot \sqrt{7}$

Opgave 5:

a. $a\sqrt{8} - a\sqrt{2} = 2a\sqrt{2} - a\sqrt{2} = a\sqrt{2}$

b. $\sqrt{2a^2} + \sqrt{\frac{1}{2} a^2} = |a| \cdot \sqrt{2} + |a| \cdot \sqrt{\frac{1}{2}} = |a| \cdot \sqrt{2} + |a| \cdot \sqrt{\frac{2}{4}} = |a| \cdot \sqrt{2} + \frac{1}{2} \cdot |a| \cdot \sqrt{2} = 1\frac{1}{2} \cdot |a| \cdot \sqrt{2}$

c. $\sqrt{24\frac{1}{2} a^2} - \sqrt{2a^2} = \sqrt{12\frac{1}{4} a^2 \cdot 2} - \sqrt{2a^2} = 3\frac{1}{2} \cdot |a| \cdot \sqrt{2} - |a| \cdot \sqrt{2} = 2\frac{1}{2} \cdot |a| \cdot \sqrt{2}$

d. $a^2 \cdot \sqrt{50} - a^2 \cdot \sqrt{32} = a^2 \cdot \sqrt{25 \cdot 2} - a^2 \cdot \sqrt{16 \cdot 2} = 5a^2 \sqrt{2} - 4a^2 \sqrt{2} = a^2 \sqrt{2}$

e. $\left(\frac{1}{4} a\sqrt{2}\right)^2 + \left(\frac{3}{4} a\sqrt{2}\right)^2 = \frac{1}{16} a^2 \cdot 2 + \frac{9}{16} a^2 \cdot 2 = \frac{1}{8} a^2 + \frac{9}{8} a^2 = 1\frac{1}{4} a^2$

f. $\frac{a}{2\sqrt{3}} + \frac{a}{\sqrt{3}} = \frac{a}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} + \frac{a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{a\sqrt{3}}{6} + \frac{a\sqrt{3}}{3} = \frac{1}{6} a\sqrt{3} + \frac{1}{3} a\sqrt{3} = \frac{1}{2} a\sqrt{3}$

Opgave 6:

I: niet waar, $\left(\sqrt{2} + \sqrt{3}\right)^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

II: waar

III: waar

IV: niet waar, $\left(10 - \sqrt{3}\right)^2 = 100 - 20\sqrt{3} + 3 = 103 - 20\sqrt{3}$

Opgave 7:

a. $\left(3\sqrt{2} - \sqrt{5}\right)^2 = 18 - 6\sqrt{10} + 5 = 23 - 6\sqrt{10}$

b. $\left(2\sqrt{2} + 3\sqrt{3}\right)^2 = 8 + 12\sqrt{6} + 27 = 35 + 12\sqrt{6}$

c. $\left(5\sqrt{3} + 2\right)\left(5\sqrt{3} - 2\right) = 75 - 4 = 71$

d. $\left(a - \sqrt{3}\right)^2 = a^2 - 2a\sqrt{3} + 3$

e. $\left(a - a\sqrt{2}\right)^2 = a^2 - 2a^2 \cdot \sqrt{2} + 2a^2 = 3a^2 - 2a^2 \cdot \sqrt{2}$

f. $\left(4 - \frac{1}{2} a\sqrt{2}\right)^2 = 16 - 4a\sqrt{2} + \frac{1}{4} a^2 \cdot 2 = 16 - 4a\sqrt{2} + \frac{1}{2} a^2$

Opgave 8:

a. $\frac{2}{\sqrt{5} - 1} = \frac{2}{\sqrt{5} - 1} \cdot \frac{\sqrt{5} + 1}{\sqrt{5} + 1} = \frac{2\sqrt{5} + 2}{5 - 1} = \frac{2\sqrt{5} + 2}{4} = \frac{1}{2}\sqrt{5} + \frac{1}{2}$

b. $\frac{10}{\sqrt{2} + \sqrt{3}} = \frac{10}{\sqrt{2} + \sqrt{3}} \cdot \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{10\sqrt{2} - 10\sqrt{3}}{2 - 3} = \frac{10\sqrt{2} - 10\sqrt{3}}{-1} = -10\sqrt{2} + 10\sqrt{3}$

c. $\frac{12\sqrt{2}}{\sqrt{10} - \sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{10} - \sqrt{2}} \cdot \frac{\sqrt{10} + \sqrt{2}}{\sqrt{10} + \sqrt{2}} = \frac{12\sqrt{20} + 24}{10 - 2} = \frac{24\sqrt{5} + 24}{8} = 3\sqrt{5} + 3$

Opgave 9:

a. $\left(2a\sqrt{2} - a\sqrt{3}\right)^2 = 8a^2 - 4a^2 \cdot \sqrt{6} + 3a^2 = 11a^2 - 4a^2 \cdot \sqrt{6}$

b. $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \cdot \dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \dfrac{5+2\sqrt{10}+2}{5-2} = \dfrac{7+2\sqrt{10}}{3} = 2\tfrac{1}{3} + \tfrac{2}{3}\sqrt{10}$

c. $\left(\tfrac{1}{2}\sqrt{2}+\tfrac{3}{4}\sqrt{3}\right)^2 = \tfrac{1}{2} + \tfrac{3}{4}\sqrt{6} + \tfrac{27}{16} = 2\tfrac{3}{16} + \tfrac{3}{4}\sqrt{6}$

d. $\dfrac{\sqrt{72}}{3-\sqrt{3}} = \dfrac{6\sqrt{2}}{3-\sqrt{3}} \cdot \dfrac{3+\sqrt{3}}{3+\sqrt{3}} = \dfrac{18\sqrt{2}+6\sqrt{6}}{9-3} = \dfrac{18\sqrt{2}+6\sqrt{6}}{6} = 3\sqrt{2}+\sqrt{6}$

e. $\left(\dfrac{1}{\sqrt{2}-1}\right)^2 = \dfrac{1}{\left(\sqrt{2}-1\right)^2} = \dfrac{1}{2-2\sqrt{2}+1} = \dfrac{1}{3-2\sqrt{2}} = \dfrac{1}{3-2\sqrt{2}} \cdot \dfrac{3+2\sqrt{2}}{3+2\sqrt{2}} = \dfrac{3+2\sqrt{2}}{9-8} =$

$\dfrac{3+2\sqrt{2}}{1} = 3+2\sqrt{2}$

f. $\left(\dfrac{a}{2\sqrt{5}}+\dfrac{a}{\sqrt{5}}\right)^2 = \left(\dfrac{a}{2\sqrt{5}}+\dfrac{2a}{2\sqrt{5}}\right)^2 = \left(\dfrac{3a}{2\sqrt{5}}\right)^2 = \dfrac{9a^2}{20} = \tfrac{9}{20}a^2$

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4.2 Gebroken vormen

Opgave 10:

I: niet waar, $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{y}{xy} + \dfrac{x}{xy} = \dfrac{x+y}{xy}$

II: waar, $\dfrac{1}{x} - \dfrac{1}{y} = \dfrac{y}{xy} - \dfrac{x}{xy} - \dfrac{y-x}{xy}$

III: niet waar, $\dfrac{3}{2x} + \dfrac{2}{3x} = \dfrac{9}{6x} + \dfrac{4}{6x} = \dfrac{13}{6x}$

IV: waar, $\dfrac{3}{2x} - \dfrac{2}{3x} = \dfrac{9}{6x} - \dfrac{4}{6x} = \dfrac{5}{6x}$

Opgave 11:

a. $\dfrac{1}{2x} + \dfrac{2}{x} = \dfrac{1}{2x} + \dfrac{4}{2x} = \dfrac{5}{2x}$

b. $\dfrac{3}{2a} - \dfrac{2}{3a} = \dfrac{9}{6a} - \dfrac{4}{6a} = \dfrac{5}{6a}$

c. $\dfrac{1}{ab} - \dfrac{1}{b} = \dfrac{1}{ab} - \dfrac{a}{ab} = \dfrac{1-a}{ab}$

d. $\dfrac{a}{b} - \dfrac{1}{a} = \dfrac{a^2}{ab} - \dfrac{b}{ab} = \dfrac{a^2 - b}{ab}$

e. $2 + \dfrac{1}{x} = \dfrac{2x}{x} + \dfrac{1}{x} = \dfrac{2x+1}{x}$

f. $3a - \dfrac{2}{a} = \dfrac{3a^2}{a} - \dfrac{2}{a} = \dfrac{3a^2 - 2}{a}$

Opgave 12:

a. $\dfrac{1}{x} + \dfrac{1}{x+2} = \dfrac{x+2}{x(x+2)} + \dfrac{x}{x(x+2)} = \dfrac{2x+2}{x(x+2)}$

b. $\dfrac{1}{x+3} + \dfrac{1}{x+4} = \dfrac{x+4}{(x+3)(x+4)} + \dfrac{x+3}{(x+3)(x+4)} = \dfrac{2x+7}{(x+3)(x+4)}$

c. $\dfrac{x}{x-2} - \dfrac{1}{x+2} = \dfrac{x(x+2)}{(x-2)(x+2)} - \dfrac{x-2}{(x-2)(x+2)} = \dfrac{x^2+2x}{(x-2)(x+2)} - \dfrac{x-2}{(x-2)(x+2)} =$

$\dfrac{x^2 + x + 2}{(x-2)(x+2)}$

d. $\dfrac{x+2}{x+3} - \dfrac{x}{x-2} = \dfrac{(x+2)(x-2)}{(x+3)(x-2)} - \dfrac{x(x+3)}{(x+3)(x-2)} = \dfrac{x^2-4}{(x+3)(x-2)} - \dfrac{x^2+3x}{(x+3)(x-2)} =$

$\dfrac{-3x-4}{(x+3)(x-2)}$

e. $\dfrac{2x}{x+2} - \dfrac{3x}{x+3} = \dfrac{2x(x+3)}{(x+2)(x+3)} - \dfrac{3x(x+2)}{(x+2)(x+3)} = \dfrac{2x^2+6x}{(x+2)(x+3)} - \dfrac{3x^2+6x}{(x+2)(x+3)} =$

$$\frac{-x^2}{(x+2)(x+3)}$$

f. $\dfrac{x+2}{x+3} - \dfrac{x+3}{x+2} = \dfrac{(x+2)(x+2)}{(x+2)(x+3)} - \dfrac{(x+3)(x+3)}{(x+2)(x+3)} = \dfrac{x^2+4x+4}{(x+2)(x+3)} - \dfrac{x^2+6x+9}{(x+2)(x+3)} =$

$$\frac{-2x-5}{(x+2)(x+3)}$$

Opgave 13:

a. $\dfrac{1}{a} = b + \dfrac{1}{c} = \dfrac{bc}{c} + \dfrac{1}{c} = \dfrac{bc+1}{c}$

$a(bc+1) = 1 \cdot c$

$a = \dfrac{c}{bc+1}$

b. $\dfrac{1}{p} = 2q - \dfrac{1}{q} = \dfrac{2q^2}{q} - \dfrac{1}{q} = \dfrac{2q^2-1}{q}$

$p = \dfrac{q}{2q^2-1}$

c. $\dfrac{3}{y} = x - \dfrac{x}{x-1} = \dfrac{x(x-1)}{x-1} - \dfrac{x}{x-1} = \dfrac{x^2-x}{x-1} - \dfrac{x}{x-1} = \dfrac{x^2-2x}{x-1}$

$\dfrac{y}{3} = \dfrac{x-1}{x^2-2x}$

$y = \dfrac{3x-3}{x^2-2x}$

Opgave 14:

a. $\dfrac{3}{x^2y} - \dfrac{2}{xy^2} = \dfrac{3y}{x^2y^2} - \dfrac{2x}{x^2y^2} = \dfrac{3y-2x}{x^2y^2}$

b. $2x - \dfrac{x^2}{x+1} = \dfrac{2x(x+1)}{x+1} - \dfrac{x^2}{x+1} = \dfrac{2x^2+2x}{x+1} - \dfrac{x^2}{x+1} = \dfrac{x^2+2x}{x+1}$

c. $\dfrac{5a}{3b} - \dfrac{a}{b+1} = \dfrac{5a(b+1)}{3b(b+1)} - \dfrac{3ab}{3b(b+1)} = \dfrac{5ab+5a}{3b(b+1)} - \dfrac{3ab}{3b(b+1)} = \dfrac{2ab+5a}{3b(b+1)}$

d. $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} = \dfrac{x^2}{x^3} + \dfrac{x}{x^3} + \dfrac{1}{x^3} = \dfrac{x^2+x+1}{x^3}$

e. $\dfrac{2x+1}{x+1} - \dfrac{x-1}{x+2} = \dfrac{(2x+1)(x+2)}{(x+1)(x+2)} - \dfrac{(x-1)(x+1)}{(x+1)(x+2)} = \dfrac{2x^2+5x+2}{(x+1)(x+2)} - \dfrac{x^2-1}{(x+1)(x+2)} =$

$$\frac{x^2+5x+3}{(x+1)(x+2)}$$

f. $\dfrac{p^2}{p+1} - \dfrac{p^3}{p+2} = \dfrac{p^2(p+2)}{(p+1)(p+2)} - \dfrac{p^3(p+1)}{(p+1)(p+2)} = \dfrac{p^3+2p^2}{(p+1)(p+2)} - \dfrac{p^4+p^3}{(p+1)(p+2)} =$

$$\frac{-p^4+2p^2}{(p+1)(p+2)}$$

Opgave 15:

a. $\dfrac{1}{2} + \dfrac{1}{x+1} = \dfrac{x+1}{x+4}$

$\dfrac{x+1}{2(x+1)} + \dfrac{2}{2(x+1)} = \dfrac{x+1}{x+4}$

$\dfrac{x+3}{2(x+1)} = \dfrac{x+1}{x+4}$

$2(x+1)(x+1) = (x+3)(x+4)$

$2(x^2 + 2x + 1) = x^2 + 7x + 12$

$2x^2 + 4x + 2 = x^2 + 7x + 12$

$x^2 - 3x - 10 = 0$

$(x+2)(x-5) = 0$

$x = -2 \quad \vee \quad x = 5$

b. $x + 1 + \dfrac{1}{x-1} = \dfrac{x}{x+3}$

$\dfrac{(x+1)(x-1)}{x-1} + \dfrac{1}{x-1} = \dfrac{x}{x+3}$

$\dfrac{x^2 - 1}{x-1} + \dfrac{1}{x-1} = \dfrac{x}{x+3}$

$\dfrac{x^2}{x-1} = \dfrac{x}{x+3}$

$x^2(x+3) = x(x-1)$

$x^3 + 3x^2 = x^2 - x$

$x^3 + 2x^2 + x = 0$

$x(x^2 + 2x + 1) = 0$

$x(x+1)(x+1) = 0$

$x = 0 \quad \vee \quad x = -1$

c. $\dfrac{1}{x} + \dfrac{1}{x-3} = \dfrac{3}{x+1}$

$\dfrac{x-3}{x(x-3)} + \dfrac{x}{x(x-3)} = \dfrac{3}{x+1}$

$\dfrac{2x-3}{x(x-3)} = \dfrac{3}{x+1}$

$3x(x-3) = (2x-3)(x+1)$

$3x^2 - 9x = 2x^2 - x - 3$

$x^2 - 8x + 3 = 0$

$x = \dfrac{8 \pm \sqrt{64-12}}{2} = \dfrac{8 \pm \sqrt{52}}{2} = \dfrac{8 \pm 2\sqrt{13}}{2}$

$x = 4 + \sqrt{13} \quad \vee \quad x = 4 - \sqrt{13}$

Opgave 16:

I: waar, $\dfrac{2x^2+1}{x} = \dfrac{2x^2}{x} + \dfrac{1}{x} = 2x + \dfrac{1}{x}$

II: niet waar, $\dfrac{x^2-1}{x} = \dfrac{x^2}{x} - \dfrac{1}{x} = x - \dfrac{1}{x}$

III: waar, $\dfrac{x^2-1}{x^2+2x+1} = \dfrac{(x-1)(x+1)}{(x+1)(x+1)} = \dfrac{x-1}{x+1}$

Opgave 17:

a. $\dfrac{x^2-9}{x^2+6x+9} = \dfrac{(x-3)(x+3)}{(x+3)(x+3)} = \dfrac{x-3}{x+3}$

b. $\dfrac{x^2-5x}{x^2-x-20} = \dfrac{x(x-5)}{(x+4)(x-5)} = \dfrac{x}{x+4}$

c. $\dfrac{a^2-4a}{a^2+a} = \dfrac{a(a-4)}{a(a+1)} = \dfrac{a-4}{a+1}$

d. $\dfrac{a^2-4a-5}{a^3+a^2} = \dfrac{(a-5)(a+1)}{a^2(a+1)} = \dfrac{a-5}{a^2}$

e. $\dfrac{x^3-11x^2+30x}{x^2-10x+25} = \dfrac{x(x^2-11x+30)}{(x-5)(x-5)} = \dfrac{x(x-5)(x-6)}{(x-5)(x-5)} = \dfrac{x(x-6)}{x-5}$

f. $\dfrac{x^2+6x+5}{2x+2} = \dfrac{(x+1)(x+5)}{2(x+1)} = \dfrac{x+5}{2} = \tfrac{1}{2}x + 2\tfrac{1}{2}$

Opgave 18:

a. $A = \dfrac{p^2+p}{p^2-1} = \dfrac{p(p+1)}{(p-1)(p+1)} = \dfrac{p}{p-1}$

b. $T = \dfrac{t^3+4t^2}{t^2-16} = \dfrac{t^2(t+4)}{(t-4)(t+4)} = \dfrac{t^2}{t-4}$

c. $N = \dfrac{a^4+a^2-2}{a^4+3a^2+2} = \dfrac{(a^2+2)(a^2-1)}{(a^2+2)(a^2+1)} = \dfrac{a^2-1}{a^2+1}$

Opgave 19:

a. $\dfrac{4x^2+7}{x} = \dfrac{4x^2}{x} + \dfrac{7}{x} = 4x + \dfrac{7}{x}$

b. $\dfrac{a^2-2a+6}{2a} = \dfrac{a^2}{2a} - \dfrac{2a}{2a} + \dfrac{6}{2a} = \tfrac{1}{2}a - 1 + \dfrac{3}{a}$

c. $\dfrac{p^3-3p^2+2}{2p} = \dfrac{p^3}{2p} - \dfrac{3p^2}{2p} + \dfrac{2}{2p} = \tfrac{1}{2}p^2 - 1\tfrac{1}{2}p + \dfrac{1}{p}$

Opgave 20:

a. $F = \dfrac{a^2+2a-3}{a-1} + \dfrac{a^2+1}{a} = \dfrac{(a+3)(a-1)}{a-1} + \dfrac{a^2}{a} + \dfrac{1}{a} = a+3+a+\dfrac{1}{a} = 2a+3+\dfrac{1}{a}$

b. $R = \dfrac{m^4 - 4}{m^4 + 2m^2} + \dfrac{m^2 + 6}{2m^2} = \dfrac{(m^2 + 2)(m^2 - 2)}{m^2(m^2 + 2)} + \dfrac{m^2}{2m^2} + \dfrac{6}{2m^2} = \dfrac{m^2 - 2}{m^2} + \tfrac{1}{2} + \dfrac{3}{m^2} =$

$\qquad \dfrac{m^2}{m^2} - \dfrac{2}{m^2} + \tfrac{1}{2} + \dfrac{3}{m^2} = 1 + \dfrac{1}{m^2} + \tfrac{1}{2} = 1\tfrac{1}{2} + \dfrac{1}{m^2}$

c. $H = \dfrac{c^3 + 4c^2 + 1}{2c^2} - \dfrac{c^2 - 5c + 6}{2c - 6} = \dfrac{c^3}{2c^2} + \dfrac{4c^2}{2c^2} + \dfrac{1}{2c^2} - \dfrac{(c - 2)(c - 3)}{2(c - 3)} = \tfrac{1}{2}c + 2 + \dfrac{1}{2c^2} - \dfrac{c - 2}{2}$

$\qquad = \tfrac{1}{2}c + 2 + \dfrac{1}{2c^2} - \dfrac{c}{2} + \dfrac{2}{2} = \tfrac{1}{2}c + 2 + \dfrac{1}{2c^2} - \tfrac{1}{2}c + 1 = 3 + \dfrac{1}{2c^2}$

<u>Opgave 21:</u>

a. $\dfrac{x^2 + 4x + 4}{x^2 - 4} = \dfrac{10}{x - 2}$

$\dfrac{(x + 2)(x + 2)}{(x - 2)(x + 2)} = \dfrac{10}{x - 2}$

$\dfrac{x + 2}{x - 2} = \dfrac{10}{x - 2}$

$x + 2 = 10$

$x = 8$

b. $\dfrac{x^2 - 9x + 14}{x^2 + x - 6} = \dfrac{3 - x}{2x - 6}$

$\dfrac{(x - 7)(x - 2)}{(x + 3)(x - 2)} = \dfrac{-(x - 3)}{2(x - 3)}$

$\dfrac{x - 7}{x + 3} = -\dfrac{1}{2}$

$2(x - 7) = -(x + 3)$

$2x - 14 = -x - 3$

$3x = 11$

$x = 3\tfrac{2}{3}$

c. $\dfrac{x^2 - 6}{x - 3} = \dfrac{x^2 - 4}{x^2 - x - 2}$

$\dfrac{x^2 - 6}{x - 3} = \dfrac{(x - 2)(x + 2)}{(x - 2)(x + 1)}$

$\dfrac{x^2 - 6}{x - 3} = \dfrac{x + 2}{x + 1}$

$(x^2 - 6)(x + 1) = (x + 2)(x - 3)$

$x^3 + x^2 - 6x - 6 = x^2 - x - 6$

$x^3 - 5x = 0$

$x(x^2 - 5) = 0$

$x = 0 \quad \vee \quad x^2 = 5$

$x = 0 \quad \vee \quad x = -\sqrt{5} \quad \vee \quad x = \sqrt{5}$

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4.3 Machten met gehele en gebroken exponenten

Opgave 22:

I: niet waar, $x^2 \cdot x^3 = x^5$

II: waar

III: niet waar, $(2x)^3 = 2^3 \cdot x^3 = 8x^3$

IV: waar

Opgave 23:

a. $2a^3 \cdot 4a^7 = 8a^{10}$

b. $2a^3 + 4a^7 - a^3 = a^3 + 4a^7$

c. $a^6 \cdot \dfrac{1}{a^4} = \dfrac{a^6}{a^4} = a^2$

d. $(3ab^2)^4 = 3^4 a^4 (b^2)^4 = 81a^4 b^8$

e. $(5a^3)^3 \cdot 2b^7 = 5^3 (a^3)^3 \cdot 2b^7 = 125a^9 \cdot 2b^7 = 250a^9 b^7$

f. $\dfrac{15a^{18}}{3a^6} = 5a^{12}$

g. $(-2a)^3 \cdot 3a^3 = (-2)^3 a^3 \cdot 3a^3 = -8a^3 \cdot 3a^3 = -24a^6$

h. $(-2a)^2 + 3a^2 = (-2)^2 a^2 + 3a^2 = 4a^2 + 3a^2 = 7a^2$

i. $\dfrac{1}{a^8} \cdot (a^3)^4 = \dfrac{1}{a^8} \cdot a^{12} = \dfrac{a^{12}}{a^8} = a^4$

Opgave 24:

a. $7a^3 + 5a^3 = 12a^3$

b. $7a^3 - a^3 = 6a^3$

c. $7a^5 : a^3 = 7a^2$

d. $7a^3 \cdot 5a^3 = 35a^6$

e. $(7a^3)^5 = 7^5 (a^3)^5 = 16807a^{15}$

f. $(7a)^3 + 5a^3 = 7^3 a^3 + 5a^3 = 343a^3 + 5a^3 = 348a^3$

g. $(2a)^2 + (\tfrac{1}{2}a)^2 = 2^2 a^2 + (\tfrac{1}{2})^2 a^2 = 4a^2 + \tfrac{1}{4}a^2 = 4\tfrac{1}{4}a^2$

h. $(3a)^2 - 8a^2 = 3^2 a^2 - 8a^2 = 9a^2 - 8a^2 = a^2$

i. $(\tfrac{1}{3}a)^3 - a^3 = (\tfrac{1}{3})^3 a^3 - a^3 = \tfrac{1}{27}a^3 - a^3 = -\tfrac{26}{27}a^3$

Opgave 25:

a. de grafiek van y_1 is hetzelfde als de grafiek van y_2.

b. $y = \dfrac{1}{x}$

c. $x^0 = 1$

Opgave 26:

a. $a^2 : \dfrac{1}{a^4} = a^2 : a^{-4} = a^6$

b. $a^8 : a^0 = a^8$

c. $(a^3)^{-2} = a^{-6}$

d. $\dfrac{a}{a^{12}} = a^{-11}$

e. $\dfrac{1}{a^5} : a = a^{-5} : a = a^{-6}$

f. $1 = a^0$

Opgave 27:

a. $7^{-2} = \dfrac{1}{7^2} = \dfrac{1}{49}$

b. $(\tfrac{1}{3})^{-2} = (3^{-1})^{-2} = 3^2 = 9$

c. $3 \cdot 5^{-2} = \dfrac{3}{5^2} = \dfrac{3}{25}$

d. $(\tfrac{2}{5})^{-1} = \dfrac{2^{-1}}{5^{-1}} = \dfrac{5}{2} = 2\tfrac{1}{2}$

e. $4 \cdot 10^{-3} = 4 \cdot \dfrac{1}{10^3} = \dfrac{4}{1000} = \dfrac{1}{250}$

f. $\tfrac{1}{2} : 6^{-2} = \dfrac{2^{-1}}{6^{-2}} = \dfrac{6^2}{2^1} = \dfrac{36}{2} = 18$

Opgave 28:

a. $6a^{-5} \cdot b^3 = \dfrac{6}{a^5} \cdot b^3 = \dfrac{6b^3}{a^5}$

b. $\tfrac{1}{3} a^{-3} = \dfrac{1}{3a^3}$

c. $3a^{-4} = \dfrac{3}{a^4}$

d. $(\tfrac{1}{2} a)^{-3} = (\tfrac{1}{2})^{-3} a^{-3} = (2^{-1})^{-3} a^{-3} = 2^3 a^{-3} = \dfrac{8}{a^3}$

e. $-4 \cdot (\tfrac{2}{3} a)^{-2} = -4 \cdot (\tfrac{2}{3})^{-2} a^{-2} = -4 \cdot \dfrac{2^{-2}}{3^{-2}} a^{-2} = -4 \cdot \dfrac{3^2}{2^2} a^{-2} = -4 \cdot \dfrac{9}{4} a^{-2} = -9a^{-2} = -\dfrac{9}{a^2}$

f. $(3a)^{-2} \cdot b^{-3} = 3^{-2} a^{-2} b^{-3} = \dfrac{1}{3^2 a^2 b^3} = \dfrac{1}{9a^2 b^3}$

Opgave 29:

a. $y_1 = y_3$

Opgave 30:

a. $5a^{\frac{1}{3}} = 5 \cdot \sqrt[3]{a}$

b. $2a^{-\frac{1}{4}} \cdot b = \dfrac{2}{a^{\frac{1}{4}}} \cdot b = \dfrac{2b}{\sqrt[4]{a}}$

c. $3a^{-\frac{2}{3}} = \dfrac{3}{a^{\frac{2}{3}}} = \dfrac{3}{\sqrt[3]{a^2}}$

d. $a^{-3} \cdot b^{\frac{1}{3}} = \dfrac{1}{a^3} \cdot \sqrt[3]{b} = \dfrac{\sqrt[3]{b}}{a^3}$

e. $\frac{1}{5}a^{-\frac{1}{2}} \cdot b^{\frac{1}{5}} = \dfrac{1}{5a^{\frac{1}{2}}} \cdot \sqrt[5]{b} = \dfrac{\sqrt[5]{b}}{5\sqrt{a}}$

f. $(5a)^{-\frac{1}{2}} = \dfrac{1}{(5a)^{\frac{1}{2}}} = \dfrac{1}{\sqrt{5a}}$

Opgave 31:

a. $a \cdot \sqrt[3]{a} = a \cdot a^{\frac{1}{3}} = a^{1\frac{1}{3}}$

b. $\dfrac{1}{\sqrt{a}} = \dfrac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$

c. $\dfrac{1}{a\sqrt{a}} = \dfrac{1}{a \cdot a^{\frac{1}{2}}} = \dfrac{1}{a^{1\frac{1}{2}}} = a^{-1\frac{1}{2}}$

d. $\dfrac{1}{\sqrt[4]{a^3}} = \dfrac{1}{a^{\frac{3}{4}}} = a^{-\frac{3}{4}}$

e. $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{2\frac{1}{2}}$

f. $\sqrt[3]{\dfrac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$

g. $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$

h. $a^4 \cdot \sqrt[3]{a} = a^4 \cdot a^{\frac{1}{3}} = a^{4\frac{1}{3}}$

i. $\dfrac{a^3}{\sqrt[3]{a}} = \dfrac{a^3}{a^{\frac{1}{3}}} = a^{2\frac{2}{3}}$

Opgave 32:

a. $\dfrac{x^6}{x^2 \cdot \sqrt{x}} = \dfrac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{x^6}{x^{2\frac{1}{2}}} = x^{3\frac{1}{2}}$

b. $x \cdot \sqrt[7]{x^3} = x \cdot x^{\frac{3}{7}} = x^{1\frac{3}{7}}$

c. $\dfrac{x}{\sqrt[5]{x}} = \dfrac{x}{x^{\frac{1}{5}}} = x^{\frac{4}{5}}$

d. $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{4\frac{1}{2}}$

e. $\dfrac{\sqrt[3]{x}}{\sqrt{x}} = \dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{-\frac{1}{6}}$

f. $\dfrac{1}{x^2} : \sqrt{x} = x^{-2} : x^{\frac{1}{2}} = x^{-2\frac{1}{2}}$

g. $\sqrt[3]{x^2} \cdot \dfrac{1}{x^3} = x^{\frac{2}{3}} \cdot x^{-3} = x^{-2\frac{1}{3}}$

h. $x^5 \cdot \sqrt[3]{x^6} = x^5 \cdot x^{\frac{6}{3}} = x^5 \cdot x^2 = x^7$

i. $\dfrac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \dfrac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \dfrac{x^{4\frac{1}{5}}}{x^{5\frac{1}{4}}} = x^{-1\frac{1}{20}}$

Opgave 33:

a. $x^{1,6} = 50$

$x = 50^{\frac{1}{1,6}} = 11{,}531$

b. $x^{-4,1} = 5$

$x = 5^{\frac{1}{-4,1}} = 0{,}675$

c. $x^{-1,3} = 11$

$x = 11^{\frac{1}{-1,3}} = 0{,}158$

d. $x^{-1} = 21$

$x = 21^{\frac{1}{-1}} = 0{,}048$

e. $x^{0,55} = 18$

$x = 18^{\frac{1}{0,55}} = 191{,}564$

f. $\sqrt[3]{x^2} = 28$

$x^{\frac{2}{3}} = 28$

$x = 28^{\frac{1}{2/3}} = 148{,}162$

Opgave 34:

a. $3x^{2,25} + 1 = 27$

$3x^{2,25} = 26$

$x^{2,25} = \frac{26}{3}$

$x = \left(\frac{26}{3}\right)^{\frac{1}{2,25}} = 2{,}611$

b. $5x^{-1,3} + 8 = 21$

$5x^{-1,3} = 13$

$x^{-1,3} = 2{,}6$

$x = 2{,}6^{\frac{1}{-1,3}} = 0{,}480$

c. $4x^{-1,8} + 16 = 5000$

$4x^{-1,8} = 4984$

$x^{-1,8} = 1246$

$x = 1246^{\frac{1}{-1,8}} = 0{,}019$

d. $8 - 3x^{1,16} = 1$

$-3x^{1,16} = -7$

$x^{1,16} = \frac{7}{3}$

$x = \left(\frac{7}{3}\right)^{\frac{1}{1,16}} = 2{,}076$

e. $5 \cdot \sqrt[3]{x} = 8$

$\sqrt[3]{x} = 1,6$

$x = 1,6^3 = 4,096$

f. $3 \cdot \sqrt[4]{x^3} - 1 = 36$

$3 \cdot \sqrt[4]{x^3} = 37$

$\sqrt[4]{x^3} = \frac{37}{3}$

$x^{\frac{3}{4}} = \frac{37}{3}$

$x = \left(\frac{37}{3}\right)^{\frac{1}{3\frac{3}{4}}} = 28,495$

Opgave 35:

a. $P = 800 \cdot l^{-2,25} = \dfrac{800}{l^{2,25}}$

Als l groter wordt, dan wordt $l^{2,25}$ groter, dus je deelt door een groter getal, dus P wordt kleiner.

b. $P = 800 \cdot 0,9^{-2,25} = 1014$ per km²

c. $800 \cdot l^{-2,25} = 1350$

$l^{-2,25} = 1,6875$

$l = 1,6875^{\frac{1}{-2,25}} = 0,79 \, \text{m}$

d. $P = 800 \cdot 2,15^{-2,25} = 142,9$

$142,9 \cdot 250 = 35731$

e. $\dfrac{160000}{5} = 32000$

$800 \cdot l^{-2,25} = 32000$

$l^{-2,25} = 40$

$l = 40^{\frac{1}{-2,25}} = 0,19 \, \text{m}$

Opgave 36:

a. $T = a \cdot R^{1,5}$

$a = \dfrac{T}{R^{1,5}} = \dfrac{1,9}{2,95^{1,5}} = 0,37$

b. $T = 0,37 \cdot 35,6^{1,5} = 79 \, \text{dagen}$

c. $0,37 \cdot R^{1,5} = \frac{15}{24}$

$R^{1,5} = 1,69$

$R = 1,69^{\frac{1}{1,5}} = 1,42$ dus $1,42 \cdot 10^5 \, \text{km}$

d. $\left(\frac{25}{11}\right)^{1,5} = 3,4$ dus 3,4 keer zo groot

Opgave 37:

a. $W = a \cdot m^{0,75}$

$a = \dfrac{W}{m^{0,75}} = \dfrac{6700}{40^{0,75}} = 421$

b. $W = 421 \cdot 4^{0,75} = 1191 \, \text{kJ}$

c. $421 \cdot m^{0,75} = 50000$

$m^{0,75} = 118,8$

$m = 118,8^{\frac{1}{0,75}} = 584$

$m = 118,8^{\frac{1}{0,75}}$

HOOFDSTUK 4: Algebra en meetkunde.

4.4 Goniometrische verhoudingen

Opgave 38:
Voor AB heb je de cosinus nodig en voor BC de sinus.

Opgave 39:
de tangens

Opgave 40:
a. $\tan \angle A = \frac{3}{5}$
   $\angle A = 31°$
b. $\sin \angle B = \frac{8}{11}$
   $\angle B = 47°$
c. $\sin \angle G = \frac{4}{10}$
   $\angle G = 24°$
d. $\tan \angle MKL = \frac{7}{10}$
   $\angle MKL = 35°$
e. $\cos \angle P = \frac{7,5}{10}$
   $\angle P = 41°$
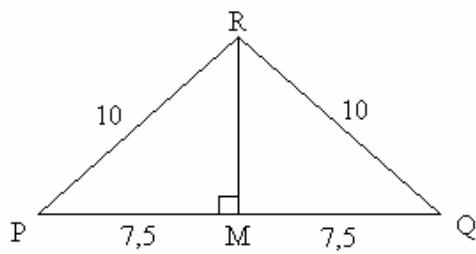
Opgave 41:
a. $\cos 38° = \dfrac{AC}{17}$
   $AC = 17 \cos 38° = 13,4$
b. $\tan 55° = \dfrac{DF}{5}$
   $DF = 5 \tan 55° = 7,1$
c. $\sin 40° = \dfrac{7}{GH}$
   $GH = \dfrac{7}{\sin 40°} = 10,9$
d. $\angle KLN = 30°$
   $\cos 30° = \dfrac{KL}{17}$
   $KL = 17 \cos 30° = 14,7$
e. $PS = 3$
   $\tan 75° = \dfrac{RS}{3}$
   $RS = 3 \tan 75° = 11,2$

Opgave 42:

$\angle AMB = \dfrac{360}{5} = 72°$
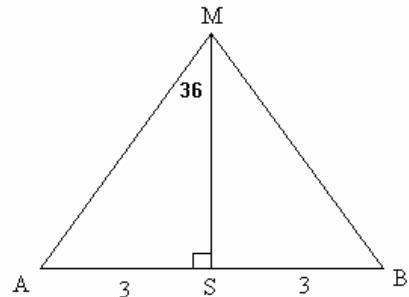
$\angle AMS = 36°$

$\tan 36° = \dfrac{3}{MS}$

$MS = \dfrac{3}{\tan 36°} = 4,13$

$Opp(\Delta ABM) = \tfrac{1}{2} \cdot 6 \cdot 4,13 = 12,39$

$Opp(ABCDE) = 5 \cdot Opp(\Delta ABM) = 5 \cdot 12,39 = 61,94$

Opgave 43:

a. $CD = \sqrt{AC^2 - AD^2} = \sqrt{(2a)^2 - a^2} = \sqrt{4a^2 - a^2} = \sqrt{3a^2} = a\sqrt{3}$

b. $\cos 60° = \dfrac{AD}{AC} = \dfrac{a}{2a} = \tfrac{1}{2}$

$\sin 60° = \dfrac{CD}{AC} = \dfrac{a\sqrt{3}}{2a} = \dfrac{\sqrt{3}}{2} = \tfrac{1}{2}\sqrt{3}$

$\tan 60° = \dfrac{CD}{AD} = \dfrac{a\sqrt{3}}{a} = \sqrt{3}$

c. $\angle ACD = \tfrac{1}{2}\angle ACB = \tfrac{1}{2} \cdot 60° = 30°$

d. $\sin 30° = \dfrac{AD}{AC} = \dfrac{a}{2a} = \tfrac{1}{2}$

$\cos 30° = \dfrac{CD}{AC} = \dfrac{a\sqrt{3}}{2a} = \dfrac{\sqrt{3}}{2} = \tfrac{1}{2}\sqrt{3}$

$\tan 30° = \dfrac{AD}{CD} = \dfrac{a}{a\sqrt{3}} = \dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} = \tfrac{1}{3}\sqrt{3}$

Opgave 44:

Stel $CD = x$

dan is $BD = x$

$\tan 60° = \dfrac{CD}{AD} = \dfrac{x}{AD}$
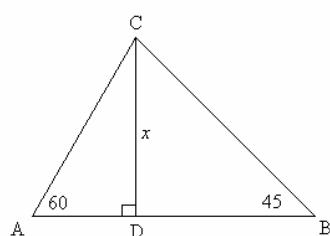
$AD = \dfrac{x}{\tan 60°} = \dfrac{x}{\sqrt{3}}$

$AD + BD = \dfrac{x}{\sqrt{3}} + x = 12$

$x \cdot \left(\tfrac{1}{\sqrt{3}} + 1\right) = 12$

$x = \dfrac{12}{\left(\tfrac{1}{\sqrt{3}}+1\right)} = \dfrac{12}{\tfrac{1}{\sqrt{3}}+\tfrac{\sqrt{3}}{\sqrt{3}}} = \dfrac{12}{\tfrac{1+\sqrt{3}}{\sqrt{3}}} = \dfrac{12\sqrt{3}}{1+\sqrt{3}} = \dfrac{12\sqrt{3}}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{12\sqrt{3}-36}{1-3} = \dfrac{12\sqrt{3}-36}{-2} = -6\sqrt{3}+18$

$Opp(\Delta ABC) = \tfrac{1}{2} \cdot AB \cdot CD = \tfrac{1}{2} \cdot 12 \cdot (18 - 6\sqrt{3}) = 108 - 36\sqrt{3}$

Opgave 45:

In ΔASK geldt: $\cos 30° = \dfrac{SK}{4}$

$SK = 4\cos 30° = 4 \cdot \tfrac{1}{2}\sqrt{3} = 2\sqrt{3}$ dus $PK = 4\sqrt{3}$

In ΔKSQ geldt: $\tan 60° = \dfrac{QS}{SK} = \dfrac{QS}{2\sqrt{3}}$

$QS = 2\sqrt{3} \cdot \tan 60° = 2\sqrt{3} \cdot \sqrt{3} = 6$

$QK = \sqrt{QS^2 + SK^2} = \sqrt{6^2 + (2\sqrt{3})^2} = \sqrt{36 + 12} = \sqrt{48}$

$QK = \sqrt{48} = 4\sqrt{3}$
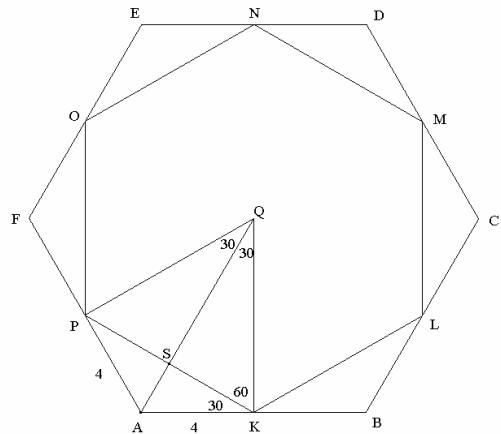
$Opp(\Delta ABQ) = \tfrac{1}{2} \cdot AB \cdot QK = \tfrac{1}{2} \cdot 8 \cdot 4\sqrt{3} = 16\sqrt{3}$

$Opp(ABCDEF) = 6 \cdot Opp(\Delta ABQ) = 6 \cdot 16\sqrt{3} = 96\sqrt{3}$

$Opp(\Delta QPK) = \tfrac{1}{2} \cdot PK \cdot QS = \tfrac{1}{2} \cdot 4\sqrt{3} \cdot 6 = 12\sqrt{3}$

$Opp(KLMNOP) = 6 \cdot Opp(\Delta QPK) = 6 \cdot 12\sqrt{3} = 72\sqrt{3}$

$Opp(gearceerde\ gebied) = Opp(ABCDEF) - Opp(KLMNOP) = 96\sqrt{3} - 72\sqrt{3} = 24\sqrt{3}$

Opgave 46:

Stel $AK = AL = x$

dan is $KL = \sqrt{x^2 + x^2} = \sqrt{2x^2} = x\sqrt{2}$
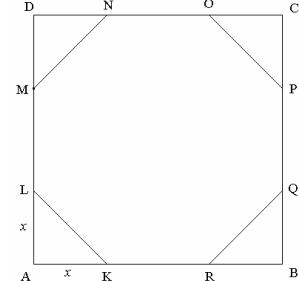
$KR = KL = x\sqrt{2}$

$AB = AK + KR + BR = x + x\sqrt{2} + x = 2x + x\sqrt{2} = 6$

$x(2 + \sqrt{2}) = 6$

$x = \dfrac{6}{2 + \sqrt{2}} = \dfrac{6}{2 + \sqrt{2}} \cdot \dfrac{2 - \sqrt{2}}{2 - \sqrt{2}} = \dfrac{12 - 6\sqrt{2}}{4 - 2} = \dfrac{12 - 6\sqrt{2}}{2} = 6 - 3\sqrt{2}$

$KL = x \cdot \sqrt{2} = (6 - 3\sqrt{2}) \cdot \sqrt{2} = 6\sqrt{2} - 6$

Opgave 47:

a. $AE = BE = a$

$AB = \sqrt{AE^2 + BE^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

In ΔBDE geldt:

$\tan 60° = \dfrac{DE}{BE} = \dfrac{DE}{a}$

$DE = a \cdot \tan 60° = a\sqrt{3}$

dus $AD = AE + DE = a + a\sqrt{3}$
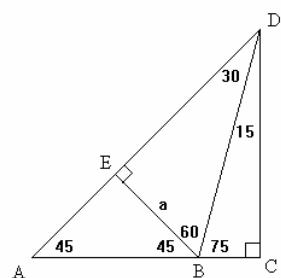
In ΔACD geldt:

$\cos 45° = \dfrac{CD}{AD} = \dfrac{CD}{a + a\sqrt{3}}$

$CD = \cos 45° \cdot (a + a\sqrt{3}) = \tfrac{1}{2}\sqrt{2} \cdot (a + a\sqrt{3}) = \tfrac{1}{2}a\sqrt{2} + \tfrac{1}{2}a\sqrt{6}$

$BC = AC - AB = CD - AB = \tfrac{1}{2}a\sqrt{2} + \tfrac{1}{2}a\sqrt{6} - a\sqrt{2} = \tfrac{1}{2}a\sqrt{6} - \tfrac{1}{2}a\sqrt{2}$

b. $BD = \sqrt{BE^2 + DE^2} = \sqrt{a^2 + (a\sqrt{3})^2} = \sqrt{a^2 + 3a^2} = \sqrt{4a^2} = 2a$

$$\sin 15° = \frac{BC}{BD} = \frac{\tfrac{1}{2}a\sqrt{6} - \tfrac{1}{2}a\sqrt{2}}{2a} = \frac{\tfrac{1}{2}\sqrt{6} - \tfrac{1}{2}\sqrt{2}}{2} = \tfrac{1}{4}\sqrt{6} - \tfrac{1}{4}\sqrt{2}$$

$$\cos 15° = \frac{CD}{BD} = \frac{\tfrac{1}{2}a\sqrt{2} + \tfrac{1}{2}a\sqrt{6}}{2a} = \frac{\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\sqrt{6}}{2} = \tfrac{1}{4}\sqrt{2} + \tfrac{1}{4}\sqrt{6}$$

$$\tan 15° = \frac{BC}{CD} = \frac{\tfrac{1}{2}\sqrt{6} - \tfrac{1}{2}\sqrt{2}}{\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\sqrt{6}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{2} + \sqrt{6}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{2} + \sqrt{6}} \cdot \frac{\sqrt{2} - \sqrt{6}}{\sqrt{2} - \sqrt{6}} = \frac{\sqrt{12} - 6 - 2 + \sqrt{12}}{2 - 6} =$$

$$\frac{2\sqrt{12} - 8}{-4} = \frac{4\sqrt{3} - 8}{-4} = -\sqrt{3} + 2 = 2 - \sqrt{3}$$

Opgave 48:

In ΔABD geldt:
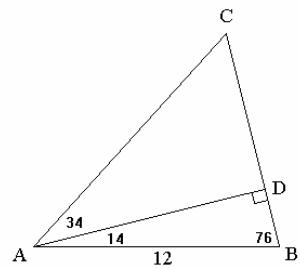
$$\sin 76° = \frac{AD}{AB} = \frac{AD}{12}$$

$$AD = 12\sin 76° = 11{,}64$$

In ΔACD geldt:

$$\cos 34° = \frac{AD}{AC} = \frac{11{,}64}{AC}$$

$$AC = \frac{11{,}64}{\cos 34°} = 14{,}04$$

Opgave 49:

In ΔACD geldt: $\sin \alpha = \dfrac{CD}{AC} = \dfrac{h}{b}$ dus $h = b \cdot \sin \alpha$

In ΔBCD geldt: $\sin \beta = \dfrac{CD}{BC} = \dfrac{h}{a}$ dus $h = a \cdot \sin \beta$
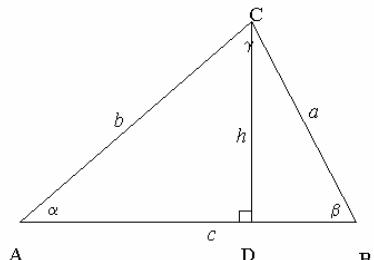
Dus $a \cdot \sin \beta = b \cdot \sin \alpha$

Dus $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta}$

Op dezelfde manier kun je bewijzen:

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}$$

Dus $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta} = \dfrac{c}{\sin \gamma}$

Opgave 50:

a. $\gamma = 180° - \alpha - \beta = 180° - 50° - 75° = 55°$

b. $\dfrac{6{,}8}{\sin 50°} = \dfrac{b}{\sin 75°} = \dfrac{c}{\sin 55°}$

$$b = \frac{6{,}8 \cdot \sin 75°}{\sin 50°} = 8{,}6$$

$$c = \frac{6{,}8 \cdot \sin 55°}{\sin 50°} = 7{,}3$$
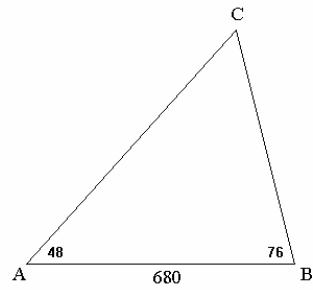
**Opgave 51:**

$\angle C = 180° - 48° - 76° = 56°$

$$\frac{AC}{\sin \angle B} = \frac{AB}{\sin \angle C}$$

$$\frac{AC}{\sin 76°} = \frac{680}{\sin 56°}$$

$$AC = \frac{680 \cdot \sin 76°}{\sin 56°} = 796 \, \text{m}$$

Opgave 52:

a.　$\sin 55° = 0,819$

b.　$\sin 125° = 0,819$

Opgave 53:

$\angle M = 180° - 20° - 110° = 50°$

$$\frac{KL}{\sin \angle M} = \frac{KM}{\sin \angle L} = \frac{LM}{\sin \angle K}$$

$$\frac{KL}{\sin 50°} = \frac{KM}{\sin 110°} = \frac{5,3}{\sin 20°}$$

$$KL = \frac{5,3 \cdot \sin 50°}{\sin 20°} = 11,9$$

$$KM = \frac{5,3 \cdot \sin 110°}{\sin 20°} = 14,6$$

Opgave 54:

a.　Teken vanuit punt A een halflijn naar rechts.
　　Teken bij punt A een hoek van $50°$.
　　Teken lijnstuk AC zo dat $AC = 6$.
　　Teken een cirkel met middelpunt C en straal 5.
　　Waar deze cirkel de halflijn snijdt ligt punt B
　　(twee mogelijkheden).

b.　$$\frac{BC}{\sin \alpha} = \frac{AC}{\sin \beta}$$

$$\frac{5}{\sin 50°} = \frac{6}{\sin \beta}$$

$$\sin \beta = \frac{6 \cdot \sin 50°}{5} = 0,919$$

$$\beta = 67°$$

$$\gamma = 180° - 50° - 67° = 63°$$
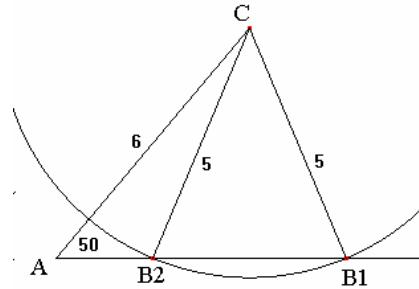
$$\frac{c}{\sin 63°} = \frac{5}{\sin 50°}$$

$$c = \frac{5 \cdot \sin 63°}{\sin 50°} = 5,8$$

c.　$\sin \beta = 0,919$

　　$\beta = 180° - 67° = 113°$

　　$\gamma = 180° - 50° - 113° = 17°$

$$\frac{c}{\sin 17°} = \frac{5}{\sin 50°}$$

$$c = \frac{5 \cdot \sin 17°}{\sin 50°} = 1,9$$

Opgave 55:
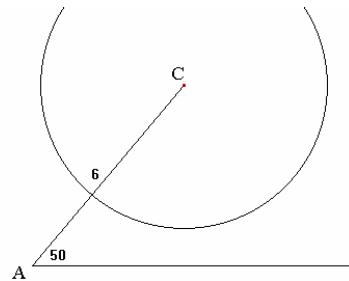
$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\frac{4}{\sin 50°} = \frac{6}{\sin \beta}$$

$$\sin \beta = \frac{6 \cdot \sin 50°}{4} = 1,15$$

β bestaat niet

Of: als je de cirkel met middelpunt C en straal 4 tekent, dan snijdt deze cirkel de halflijn niet.

Opgave 56:

Teken de cirkel met middelpunt C die de halflijn k raakt in punt R.
Voor deze waarde van a is er één cirkel.
Als de straal van de cirkel iets groter wordt dan snijdt de cirkel de halflijn in twee punten (zie ook opgave 54) zodat er twee driehoeken mogelijk zijn.
Als de straal nog groter wordt schuift het linker snijpunt naar punt A toe. Op het moment dat dit linker snijpunt samenvalt met punt A is er nog maar één driehoek. Als de straal nog groter wordt heb je nog steeds maar één driehoek.
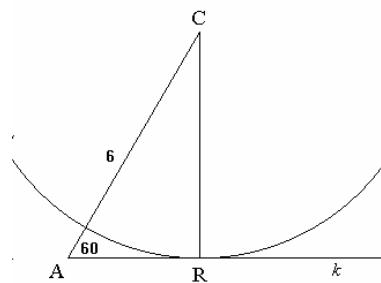
$$\sin 60° = \frac{CR}{6}$$

$$CR = 6 \cdot \sin 60° = 6 \cdot \tfrac{1}{2}\sqrt{3} = 3\sqrt{3}$$

a. geen driehoek als $a < 3\sqrt{3}$

b. één driehoek als $a = 3\sqrt{3} \quad \vee \quad a \geq 6$

c. twee driehoeken als $3\sqrt{3} < a < 6$

Opgave 57:

a. je kent alleen 3 zijden en voor de sinusregel heb je twee hoeken en een zijde of twee zijden en een hoek nodig.

b. je moet een hoek kennen tegenover een gegeven zijde of een zijde tegenover een gegeven hoek.

Opgave 58:
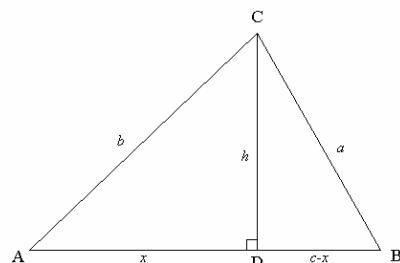
In $\triangle ACD$ geldt: $b^2 = h^2 + x^2$ dus $h^2 = b^2 - x^2$

$\cos \angle A = \dfrac{x}{b}$ dus $x = b \cdot \cos \angle A$

In $\triangle BCD$ geldt: $a^2 = h^2 + (c - x)^2$ dus $h^2 = a^2 - (c - x)^2$

Dus geldt: $a^2 - (c - x)^2 = b^2 - x^2$

$a^2 = b^2 + (c - x)^2 - x^2$

$$a^2 = b^2 + c^2 - 2 \cdot c \cdot x + x^2 - x^2$$
$$a^2 = b^2 + c^2 - 2 \cdot c \cdot b \cdot \cos \angle A$$
$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$

Opgave 59:
$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$
$$5^2 = 6^2 + 7^2 - 2 \cdot 6 \cdot 7 \cdot \cos \alpha$$
$$25 = 36 + 49 - 84 \cdot \cos \alpha$$
$$84 \cdot \cos \alpha = 60$$
$$\cos \alpha = \frac{60}{84}$$
$$\alpha = 44°$$
$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$
$$\frac{5}{\sin 44°} = \frac{6}{\sin \beta}$$
$$\sin \beta = \frac{6 \cdot \sin 44°}{5} = 0,834$$
$$\beta = 56°$$
$$\gamma = 180° - 44° - 56° = 80°$$

Opgave 60:
$$EF^2 = DE^2 + DF^2 - 2 \cdot DE \cdot DF \cdot \cos \angle D$$
$$4^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos \angle D$$
$$16 = 25 + 49 - 70 \cdot \cos \angle D$$
$$70 \cdot \cos \angle D = 58$$
$$\cos \angle D = \frac{58}{70}$$
$$\angle D = 34°$$
$$\frac{EF}{\sin \angle D} = \frac{DF}{\sin \angle E}$$
$$\frac{4}{\sin 34°} = \frac{7}{\sin \angle E}$$
$$\sin \angle E = \frac{7 \cdot \sin 34°}{4} = 0,979$$
$\angle E = 78° \quad \vee \quad \angle E = 102°$ als je de driehoek tekent zie je dat alleen de stompe hoek kan dus $\angle E = 102°$ en dan geldt $\angle F = 180° - 34° - 102° = 44°$

Opgave 61:
a. $\quad a^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cdot \cos 50° = 22,4$
$\quad\quad a = 4,74$

b. $\quad \dfrac{4,74}{\sin 50°} = \dfrac{5}{\sin \beta}$

$\quad\quad \sin \beta = \dfrac{5 \cdot \sin 50°}{4,74} = 0,808$ dus $\beta = 54°$

Opgave 62:

$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos \angle A$

$8^2 = 10^2 + 7^2 - 2 \cdot 10 \cdot 7 \cdot \cos \angle A$

$64 = 100 + 49 - 140 \cdot \cos \angle A$
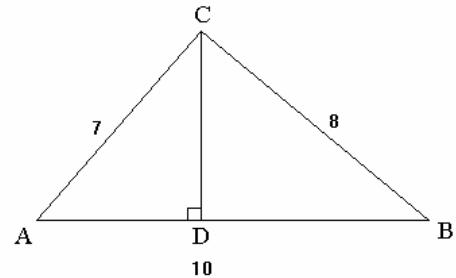
$140 \cdot \cos \angle A = 85$

$\cos \angle A = \dfrac{85}{140} = 0,607$

$\angle A = 52,6°$

$\sin 52,6° = \dfrac{CD}{7}$

$CD = 7 \cdot \sin 52,6° = 5,6$

Opgave 63:

a. In $\triangle ASB$ geldt:

$4,5^2 = 7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cdot \cos \angle BAS$

$20,25 = 49 + 100 - 140 \cdot \cos \angle BAS$

$140 \cdot \cos \angle BAS = 128,75$

$\cos \angle BAS = 0,920$

$\angle BAS = 23,1°$

In $\triangle ABC$ geldt:
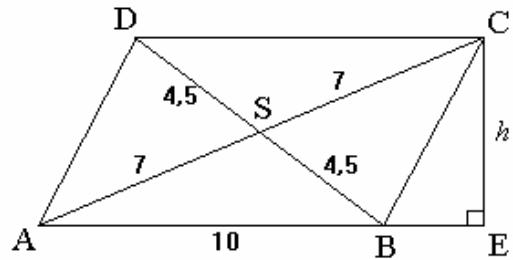
$BC^2 = 10^2 + 14^2 - 2 \cdot 10 \cdot 14 \cdot \cos 23,1° = 38,4$

$BC = 6,2$

b. In $\triangle ACE$ geldt: $\sin 23,1° = \dfrac{h}{14}$

$h = 14 \cdot \sin 23,1° = 5,49$

$Opp(ABCD) = AB \cdot h = 10 \cdot 5,49 = 54,9$

HOOFDSTUK 4: Algebra en meetkunde.

4.5 Lengte en oppervlakte
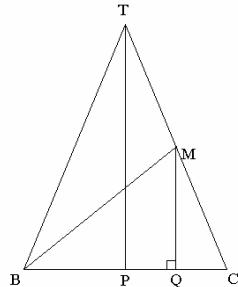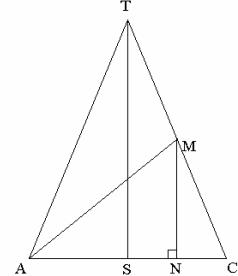
Opgave 64:
a. $BE = \sqrt{AB^2 + AE^2} = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$

b. $BH = \sqrt{BE^2 + EH^2} = \sqrt{(6\sqrt{2})^2 + 6^2} = \sqrt{72 + 36} = \sqrt{108} = 6\sqrt{3}$

Opgave 65:
a. $AF = \sqrt{AB^2 + BF^2} = \sqrt{(2a)^2 + a^2} = \sqrt{4a^2 + a^2} = \sqrt{5a^2} = a\sqrt{5}$

b. $AG = \sqrt{AF^2 + FG^2} = \sqrt{(a\sqrt{5})^2 + a^2} = \sqrt{5a^2 + a^2} = \sqrt{6a^2} = a\sqrt{6}$

c. $AC = \sqrt{AB^2 + BC^2} = \sqrt{(2a)^2 + a^2} = \sqrt{4a^2 + a^2} = \sqrt{5a^2} = a\sqrt{5}$

$AM = \sqrt{AC^2 + CM^2} = \sqrt{(a\sqrt{5})^2 + (\tfrac{1}{2}a)^2} = \sqrt{5a^2 + \tfrac{1}{4}a^2} = \sqrt{5\tfrac{1}{4}a^2} = \tfrac{1}{2}a\sqrt{21}$
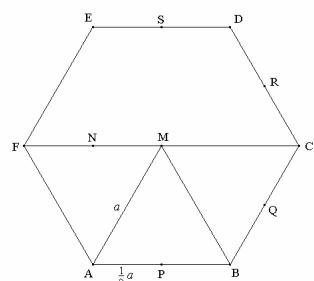
Opgave 66:
a. $AC = a\sqrt{2}$ dus $AS = \tfrac{1}{2}a\sqrt{2}$

$AT = \sqrt{AS^2 + TS^2} = \sqrt{(\tfrac{1}{2}a\sqrt{2})^2 + (2a)^2} = \sqrt{\tfrac{1}{2}a^2 + 4a^2} =$
$\sqrt{4\tfrac{1}{2}a^2} = \sqrt{\tfrac{18}{4}a^2} = \tfrac{3}{2}a\sqrt{2}$

b. $AN = \tfrac{3}{4}AC = \tfrac{3}{4}a\sqrt{2}$

$MN = \tfrac{1}{2}TS = a$

$AM = \sqrt{AN^2 + MN^2} = \sqrt{(\tfrac{3}{4}a\sqrt{2})^2 + a^2} = \sqrt{\tfrac{9}{8}a^2 + a^2} =$
$\sqrt{\tfrac{17}{8}a^2} = \sqrt{\tfrac{34}{16}a^2} = \tfrac{1}{4}a\sqrt{34}$

c. $TP = \sqrt{TS^2 + PS^2} = \sqrt{(2a)^2 + (\tfrac{1}{2}a)^2} = \sqrt{4a^2 + \tfrac{1}{4}a^2} =$
$\sqrt{4\tfrac{1}{4}a^2} = \sqrt{\tfrac{17}{4}a^2} = \tfrac{1}{2}a\sqrt{17}$

$QM = \tfrac{1}{2}TP = \tfrac{1}{4}a\sqrt{17}$

$BQ = \tfrac{3}{4}BC = \tfrac{3}{4}a$

$BM = \sqrt{BQ^2 + QM^2} = \sqrt{(\tfrac{3}{4}a)^2 + (\tfrac{1}{4}a\sqrt{17})^2} = \sqrt{\tfrac{9}{16}a^2 + \tfrac{17}{16}a^2} =$
$\sqrt{\tfrac{26}{16}a^2} = \tfrac{1}{4}a\sqrt{26}$
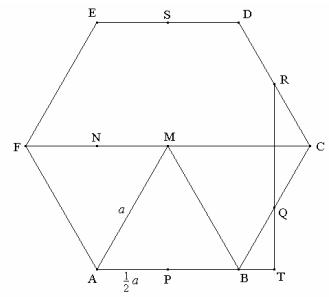
Opgave 67:
a. $PM = \sqrt{AM^2 - AP^2} = \sqrt{a^2 - (\tfrac{1}{2}a)^2} = \sqrt{a^2 - \tfrac{1}{4}a^2} =$
$\sqrt{\tfrac{3}{4}a^2} = \tfrac{1}{2}a\sqrt{3}$

$PS = 2 \cdot PM = 2 \cdot \tfrac{1}{2}a\sqrt{3} = a\sqrt{3}$

b. $AC = \sqrt{AN^2 + CN^2} = \sqrt{(\tfrac{1}{2}a\sqrt{3})^2 + (1\tfrac{1}{2}a)^2} =$
$\sqrt{\tfrac{3}{4}a^2 + 2\tfrac{1}{4}a^2} = \sqrt{3a^2} = a\sqrt{3}$

c. $AQ = \sqrt{AT^2 + QT^2} = \sqrt{(1\frac{1}{4}a)^2 + (\frac{1}{4}a\sqrt{3})^2} =$

$\quad \sqrt{\frac{25}{16}a^2 + \frac{3}{16}a^2} = \sqrt{\frac{28}{16}a^2} = \sqrt{\frac{7}{4}a^2} = \frac{1}{2}a\sqrt{7}$

$AR = \sqrt{AT^2 + RT^2} = \sqrt{(1\frac{1}{4}a)^2 + (\frac{3}{4}a\sqrt{3})^2} =$

$\quad \sqrt{\frac{25}{16}a^2 + \frac{27}{16}a^2} = \sqrt{\frac{52}{16}a^2} = \sqrt{\frac{13}{4}a^2} = \frac{1}{2}a\sqrt{13}$

$AS = \sqrt{AP^2 + PS^2} = \sqrt{(\frac{1}{2}a)^2 + (a\sqrt{3})^2} = \sqrt{\frac{1}{4}a^2 + 3a^2} =$

$\quad \sqrt{3\frac{1}{4}a^2} = \sqrt{\frac{13}{4}a^2} = \frac{1}{2}a\sqrt{13}$
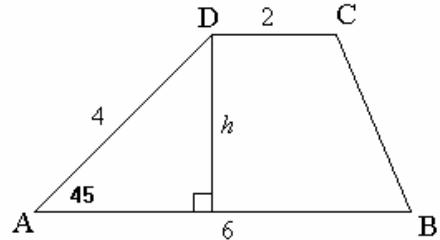
Opgave 68:

$\sin 45° = \dfrac{h}{4}$

$h = 4 \cdot \sin 45° = 4 \cdot \frac{1}{2}\sqrt{2} = 2\sqrt{2}$

$Opp(trapezium) = \frac{1}{2} \cdot h \cdot (basis_1 + basis_2)$

$Opp(ABCD) = \frac{1}{2} \cdot 2\sqrt{2} \cdot (6 + 2) = 8\sqrt{2}$

Opgave 69:

a. $\tan 60° = \dfrac{h}{AE} = \dfrac{4}{AE}$

$AE = \dfrac{4}{\tan 60°} = \dfrac{4}{\sqrt{3}} = \dfrac{4}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3} = \frac{4}{3}\sqrt{3}$

$BF = CF = h = 4$

$CD = EF = 10 - AE - BF = 10 - \frac{4}{3}\sqrt{3} - 4 = 6 - \frac{4}{3}\sqrt{3}$

$Opp(ABCD) = \frac{1}{2} \cdot 4 \cdot (10 + 6 - \frac{4}{3}\sqrt{3}) = 32 - \frac{8}{3}\sqrt{3}$

b. $AE = \dfrac{h}{\tan 60°} = \dfrac{h}{\sqrt{3}} = \dfrac{h}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{h \cdot \sqrt{3}}{3} = \frac{1}{3}h \cdot \sqrt{3}$

$BF = h$

$AE + EF + FB = 10$

$\frac{1}{3}h \cdot \sqrt{3} + 2 + h = 10$

$h \cdot (\frac{1}{3}\sqrt{3} + 1) = 8$

$h = \dfrac{8}{\frac{1}{3}\sqrt{3} + 1} = \dfrac{8}{\frac{1}{3}\sqrt{3} + 1} \cdot \dfrac{\frac{1}{3}\sqrt{3} - 1}{\frac{1}{3}\sqrt{3} - 1} = \dfrac{\frac{8}{3}\sqrt{3} - 8}{\frac{1}{3} - 1} = \dfrac{\frac{8}{3}\sqrt{3} - 8}{-\frac{2}{3}} = -4\sqrt{3} + 12 = 12 - 4\sqrt{3}$

$Opp(ABCD) = \frac{1}{2} \cdot (12 - 4\sqrt{3}) \cdot (10 + 2) = 72 - 24\sqrt{3}$

c. $AE = \frac{1}{3}h \cdot \sqrt{3}$ en $BF = h$
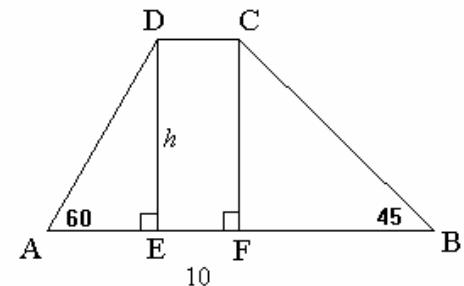
$CD = 10 - \frac{1}{3}h \cdot \sqrt{3} - h$

$Opp(ABCD) = \frac{1}{2} \cdot h \cdot (10 + 10 - \frac{1}{3}h\sqrt{3} - h) = \frac{1}{2}h \cdot (20 - \frac{1}{3}h\sqrt{3} - h) = 25$

$y_1 = \frac{1}{2}x \cdot (20 - \frac{1}{3}x\sqrt{3} - x)$ en $y_2 = 25$

calc-menu optie intersection geeft $x = 3{,}43 \quad \vee \quad x = 9{,}25$

$CD = 10 - \frac{1}{3} \cdot 3{,}43 \cdot \sqrt{3} - 3{,}43 = 4{,}60 \quad \vee \quad x = 10 - \frac{1}{3} \cdot 9{,}25 \cdot \sqrt{3} - 9{,}25 = -4{,}59$ (vervalt)
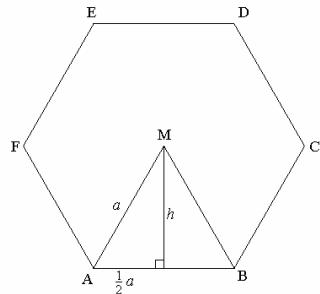
Opgave 70:

a. $h = \sqrt{a^2 - (\tfrac{1}{2}a)^2} = \sqrt{a^2 - \tfrac{1}{4}a^2} = \sqrt{\tfrac{3}{4}a^2} = \tfrac{1}{2}a\sqrt{3}$

$Opp(\Delta ABM) = \tfrac{1}{2} \cdot a \cdot \tfrac{1}{2}a\sqrt{3} = \tfrac{1}{4}a^2\sqrt{3}$

$Opp(ABCDEF) = 6 \cdot Opp(\Delta ABM) = 6 \cdot \tfrac{1}{4}a^2\sqrt{3} = 1\tfrac{1}{2}a^2\sqrt{3}$

b. $Opp(cirkel) = \pi \cdot r^2 = \pi \cdot (\tfrac{1}{2}a\sqrt{3})^2 = \pi \cdot \tfrac{3}{4}a^2 = \tfrac{3}{4}\pi \cdot a^2$

c. $Opp = 1\tfrac{1}{2}a^2\sqrt{3} - \tfrac{3}{4}\pi \cdot a^2 = 10$

$y_1 = 1\tfrac{1}{2}x^2\sqrt{3} - \tfrac{3}{4}\pi \cdot x^2$ en $y_2 = 10$

calc-menu optie intersection geeft $x = 6{,}43$

dus $a = 6{,}43$

Opgave 71:
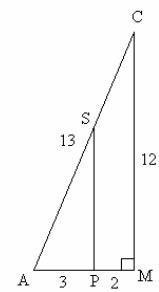
$CM = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$

snavelfiguur dus $\dfrac{AP}{AM} = \dfrac{PS}{CM}$

$\dfrac{3}{5} = \dfrac{PS}{12}$

$PS = \dfrac{3 \cdot 12}{5} = 7{,}2$

$Opp(PQRS) = 4 \cdot 7{,}2 = 28{,}8$

Opgave 72:

a. $BC = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

snavelfiguur dus $\dfrac{PQ}{AB} = \dfrac{CP}{AC} = \dfrac{CQ}{BC}$

$\dfrac{PQ}{3} = \dfrac{x}{4} = \dfrac{CQ}{5}$

$PQ = \dfrac{3x}{4} = \tfrac{3}{4}x$

$CQ = \dfrac{5x}{4} = \tfrac{5}{4}x$

$AP = 4 - CP = 4 - x$

$Opp(\Delta BPQ) = Opp(\Delta ABC) - Opp(\Delta CPQ) - Opp(\Delta ABP) =$

$\qquad \tfrac{1}{2} \cdot 3 \cdot 4 - \tfrac{1}{2} \cdot x \cdot \tfrac{3}{4}x - \tfrac{1}{2} \cdot 3 \cdot (4 - x) = 6 - \tfrac{3}{8}x^2 - 6 + 1\tfrac{1}{2}x = 1\tfrac{1}{2}x - \tfrac{3}{8}x^2$

b. $Opp' = 1\tfrac{1}{2} - \tfrac{3}{4}x = 0$

$-\tfrac{3}{4}x = -1\tfrac{1}{2}$

$x = 2$

$Opp = 1\tfrac{1}{2} \cdot 2 - \tfrac{3}{8} \cdot 2^2 = 3 - 1\tfrac{1}{2} = 1\tfrac{1}{2}$

Opgave 73:

a. $\tan 30° = \dfrac{x}{AP}$

$AP = \dfrac{x}{\tan 30°} = \dfrac{x}{\frac{1}{3}\sqrt{3}} = \dfrac{x}{\frac{1}{3}\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} =$

$\dfrac{x\sqrt{3}}{1} = x\sqrt{3}$

$\cos 30° = \dfrac{AK}{6}$

$AK = 6 \cdot \cos 30° = 6 \cdot \frac{1}{2}\sqrt{3} = 3\sqrt{3}$

$AB = 2 \cdot 3\sqrt{3} + 6 = 6\sqrt{3} + 6$

$PQ = AB - 2 \cdot AP = 6\sqrt{3} + 6 - 2 \cdot x\sqrt{3}$

$Opp(PQRS) = PQ \cdot PS = (6\sqrt{3} + 6 - 2x\sqrt{3}) \cdot x$
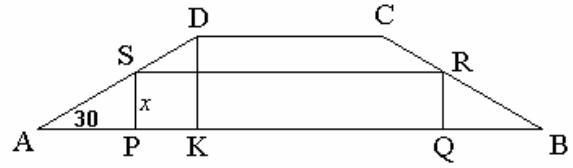
b. $Opp = (6\sqrt{3} + 6 - 2x\sqrt{3}) \cdot x = 6x\sqrt{3} + 6x - 2x^2\sqrt{3}$

$Opp' = 6\sqrt{3} + 6 - 4x\sqrt{3} = 0$

$-4x\sqrt{3} = -6\sqrt{3} - 6$

$x\sqrt{3} = 1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}$

$x = 1\frac{1}{2} + \dfrac{1\frac{1}{2}}{\sqrt{3}} = 1\frac{1}{2} + \dfrac{1\frac{1}{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = 1\frac{1}{2} + \dfrac{1\frac{1}{2}\sqrt{3}}{3} = 1\frac{1}{2} + \frac{1}{2}\sqrt{3} = PS$

Opgave 74:

a. $AE = DE = CF = BF = h$

$\cos 45° = \dfrac{h}{AD}$

$AD = \dfrac{h}{\cos 45°} = \dfrac{h}{\frac{1}{2}\sqrt{2}} = \dfrac{h}{\frac{1}{2}\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{h\sqrt{2}}{1} = h\sqrt{2}$

$Omtrek = 2h + 2h\sqrt{2} + 2x = 60$

$2x = 60 - 2h - 2h\sqrt{2}$

$x = 30 - h - h\sqrt{2}$

b. $Opp(ABCD) = \frac{1}{2}h \cdot (2h + x + x) = \frac{1}{2}h \cdot (2h + 2x) = \frac{1}{2}h \cdot (2h + 60 - 2h - 2h\sqrt{2}) =$

$\qquad \frac{1}{2}h \cdot (60 - 2h\sqrt{2}) = 30h - h^2\sqrt{2} = 0$

$Opp' = 30 - 2h\sqrt{2} = 0$

$-2h\sqrt{2} = -30$

$h = \dfrac{-30}{-2\sqrt{2}} = \dfrac{-30}{-2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{-30\sqrt{2}}{-4} = 7\frac{1}{2}\sqrt{2}$

$Opp(ABCD) = 30 \cdot 7\frac{1}{2}\sqrt{2} - (7\frac{1}{2}\sqrt{2})^2 \cdot \sqrt{2} = 225\sqrt{2} - 112\frac{1}{2}\sqrt{2} = 112\frac{1}{2}\sqrt{2}$

Opgave 75:

$MR = 6$

$RS = \sqrt{MR^2 - MS^2} = \sqrt{6^2 - x^2} = \sqrt{36 - x^2}$

$Opp(\Delta PQR) = \frac{1}{2} \cdot PQ \cdot PS = \frac{1}{2} \cdot 2\sqrt{36 - x^2} \cdot (6 + x)$

$y_1 = (6 + x) \cdot \sqrt{36 - x^2}$

calc-menu optie maximum geeft $Opp = 46{,}77$

HOOFDSTUK 4: Algebra en meetkunde.

4.6 Diagnostische toets

Opgave 1:
a. $4\sqrt{5} \cdot 3\sqrt{2} = 12\sqrt{10}$

b. $\sqrt{16\frac{1}{3}} = \sqrt{\frac{49}{3}} = \sqrt{\frac{147}{9}} = \frac{7}{3}\sqrt{3}$

c. $\dfrac{6}{\sqrt{2}} + \sqrt{8} = \dfrac{6}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} + \sqrt{8} = \dfrac{6\sqrt{2}}{2} + 2\sqrt{2} = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

d. $\sqrt{\frac{1}{3}} + \sqrt{3} = \sqrt{\frac{3}{9}} + \sqrt{3} = \frac{1}{3}\sqrt{3} + \sqrt{3} = 1\frac{1}{3}\sqrt{3}$

e. $\sqrt{8a^2} + \sqrt{32a^2} = 2 \cdot |a| \cdot \sqrt{2} + 4 \cdot |a| \cdot \sqrt{2} = 6 \cdot |a| \cdot \sqrt{2}$

f. $a\sqrt{48} - 2a\sqrt{12} = 4a\sqrt{3} - 2a \cdot 2\sqrt{3} = 4a\sqrt{3} - 4a\sqrt{3} = 0$

Opgave 2:
a. $(3 + \sqrt{2})^2 = 9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$

b. $\dfrac{\sqrt{3}}{\sqrt{5} + \sqrt{7}} = \dfrac{\sqrt{3}}{\sqrt{5} + \sqrt{7}} \cdot \dfrac{\sqrt{5} - \sqrt{7}}{\sqrt{5} - \sqrt{7}} = \dfrac{\sqrt{15} - \sqrt{21}}{5 - 7} = \dfrac{\sqrt{15} - \sqrt{21}}{-2} = -\frac{1}{2}\sqrt{15} + \frac{1}{2}\sqrt{21}$

c. $(a - \sqrt{3})(a + \sqrt{3}) = a^2 - 3$

d. $\dfrac{20}{\sqrt{6} - 1} = \dfrac{20}{\sqrt{6} - 1} \cdot \dfrac{\sqrt{6} + 1}{\sqrt{6} + 1} = \dfrac{20\sqrt{6} + 20}{6 - 1} = \dfrac{20\sqrt{6} + 20}{5} = 4\sqrt{6} + 4$

e. $(2a - \sqrt{7})^2 = 4a^2 - 4a\sqrt{7} + 7$

f. $\left(\dfrac{2}{\sqrt{5} - 1}\right)^2 = \dfrac{4}{(\sqrt{5} - 1)^2} = \dfrac{4}{5 - 2\sqrt{5} + 1} = \dfrac{4}{6 - 2\sqrt{5}} = \dfrac{4}{6 - 2\sqrt{5}} \cdot \dfrac{6 + 2\sqrt{5}}{6 + 2\sqrt{5}} = \dfrac{24 + 8\sqrt{5}}{36 - 20} =$

$\qquad \dfrac{24 + 8\sqrt{5}}{16} = 1\frac{1}{2} + \frac{1}{2}\sqrt{5}$

Opgave 3:
a. $\dfrac{1}{3a} - \dfrac{1}{4a} = \dfrac{4}{12a} - \dfrac{3}{12a} = \dfrac{1}{12a}$

b. $\dfrac{1}{5x} + \dfrac{1}{10x} = \dfrac{2}{10x} + \dfrac{1}{10x} = \dfrac{3}{10x}$

c. $\dfrac{1}{x - 2} - \dfrac{2}{x + 1} = \dfrac{x + 1}{(x - 2)(x + 1)} - \dfrac{2(x - 2)}{(x - 2)(x + 1)} = \dfrac{x + 1}{(x - 2)(x + 1)} - \dfrac{2x - 4}{(x - 2)(x + 1)} =$

$\qquad \dfrac{-x + 5}{(x - 2)(x + 1)}$

d. $\dfrac{2x}{x + 1} + \dfrac{5}{x - 3} = \dfrac{2x(x - 3)}{(x + 1)(x - 3)} + \dfrac{5(x + 1)}{(x + 1)(x - 3)} = \dfrac{2x^2 - 6x}{(x + 1)(x - 3)} + \dfrac{5x + 5}{(x + 1)(x - 3)} =$

$\qquad \dfrac{2x^2 - x + 5}{(x + 1)(x - 3)}$

e. $x + \dfrac{3}{x + 1} = \dfrac{x(x + 1)}{x + 1)} + \dfrac{3}{x + 1} = \dfrac{x^2 + x}{x + 1} + \dfrac{3}{x + 1} = \dfrac{x^2 + x + 3}{x + 1}$

f. $\dfrac{2a}{b} + \dfrac{a}{a+b} = \dfrac{2a(a+b)}{b(a+b)} + \dfrac{ab}{b(a+b)} = \dfrac{2a^2 + 2ab}{b(a+b)} + \dfrac{ab}{b(a+b)} = \dfrac{2a^2 + 3ab}{b(a+b)}$

Opgave 4:

a. $\dfrac{x^2 - 6x + 5}{x^2 - 25} = \dfrac{(x-5)(x-1)}{(x-5)(x+5)} = \dfrac{x-1}{x+5}$

b. $\dfrac{6x^2 + 6x}{x^2 + 3x + 2} = \dfrac{6x(x+1)}{(x+2)(x+1)} = \dfrac{6x}{x+2}$

c. $\dfrac{x^2 + 6x + 8}{x+2} + \dfrac{x^2 + 8}{x} = \dfrac{(x+2)(x+4)}{x+2} + \dfrac{x^2 + 8}{x} = x + 4 + x + \dfrac{8}{x} = 2x + 4 + \dfrac{8}{x}$

Opgave 5:

a. $\dfrac{6}{x} - \dfrac{4}{x+2} = 2$

$\dfrac{6(x+2)}{x(x+2)} - \dfrac{4x}{x(x+2)} = 2$

$\dfrac{6x+12}{x(x+2)} - \dfrac{4x}{x(x+2)} = 2$

$\dfrac{2x+12}{x(x+2)} = 2$

$2x(x+2) = 2x + 12$

$2x^2 + 4x = 2x + 12$

$2x^2 + 2x - 12 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3 \quad \vee \quad x = 2$

b. $\dfrac{x}{16} = \dfrac{x^2 - 4}{x^2 + 6x + 8}$

$\dfrac{x}{16} = \dfrac{(x+2)(x-2)}{(x+2)(x+4)}$

$\dfrac{x}{16} = \dfrac{x-2}{x+4}$

$x(x+4) = 16(x-2)$

$x^2 + 4x = 16x - 32$

$x^2 - 12x + 32 = 0$

$(x-4)(x-8) = 0$

$x = 4 \quad \vee \quad x = 8$

c. $\dfrac{x^2 - 9}{x^2 + 4x + 3} = \dfrac{5}{3x}$

$\dfrac{(x+3)(x-3)}{(x+3)(x+1)} = \dfrac{5}{3x}$

$\dfrac{x-3}{x+1} = \dfrac{5}{3x}$

$$3x(x-3) = 5(x+1)$$
$$3x^2 - 9x = 5x + 5$$
$$3x^2 - 14x - 5 = 0$$
$$x = \frac{14 \pm \sqrt{196 + 60}}{6} = \frac{14 \pm \sqrt{256}}{6} = \frac{14 \pm 16}{6}$$
$$x = \frac{14 + 16}{6} = 5 \quad \vee \quad x = \frac{14 - 16}{6} = -\tfrac{1}{3}$$

Opgave 6:

a. $2a^3 \cdot 3a^6 = 6a^9$

b. $a^{12} \cdot \dfrac{1}{a^4} = \dfrac{a^{12}}{a^4} = a^8$

c. $(2a)^3 - a \cdot 7a^2 = 8a^3 - 7a^3 = a^3$

d. $\dfrac{14a^8}{2a^5} = 7a^3$

e. $(3a^2)^4 + 5(a^4)^2 = 81a^8 + 5a^8 = 86a^8$

f. $\dfrac{1}{a^6} \cdot (a^2)^3 = \dfrac{1}{a^6} \cdot a^6 = \dfrac{a^6}{a^6} = 1$

Opgave 7:

a. $\dfrac{1}{a^3} = a^{-3}$

b. $a^4 \cdot \dfrac{1}{a^7} = \dfrac{a^4}{a^7} = a^{-3}$

c. $\sqrt[5]{a^3} = a^{\frac{3}{5}}$

d. $\dfrac{\sqrt{a}}{a^2} = \dfrac{a^{\frac{1}{2}}}{a^2} = a^{-1\frac{1}{2}}$

e. $a^2 \cdot \sqrt[3]{a} = a^2 \cdot a^{\frac{1}{3}} = a^{2\frac{1}{3}}$

f. $\dfrac{1}{\sqrt[3]{a^2}} = \dfrac{1}{a^{\frac{2}{3}}} = a^{-\frac{2}{3}}$

Opgave 8:

a. $\left(a^{-\frac{1}{4}}\right)^3 = a^{-\frac{3}{4}} = \dfrac{1}{a^{\frac{3}{4}}} = \dfrac{1}{\sqrt[4]{a^3}}$

b. $a^{-2} \cdot b^{\frac{1}{5}} = \dfrac{1}{a^2} \cdot \sqrt[5]{b} = \dfrac{\sqrt[5]{b}}{a^2}$

c. $7a^{-\frac{1}{3}} \cdot b^{\frac{3}{5}} = \dfrac{7 \cdot \sqrt[5]{b^3}}{a^{\frac{1}{3}}} = \dfrac{7 \cdot \sqrt[5]{b^3}}{\sqrt[3]{a}}$

Opgave 9:

a. $3x^{1,6} + 2 = 7$
   $3x^{1,6} = 5$

$$x^{1,6} = \tfrac{5}{3}$$

$$x = (\tfrac{5}{3})^{\frac{1}{1,6}} = 1,376$$

b. $\tfrac{1}{4}x^{-3,7} = 160$

$$x^{-3,7} = 640$$

$$x = 640^{\frac{1}{-3,7}} = 0,174$$

c. $7 \cdot \sqrt[5]{x^3} = 48$
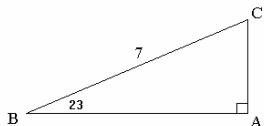
$$x^{\frac{3}{5}} = \tfrac{48}{7}$$

$$x = (\tfrac{48}{7})^{\frac{1}{3/5}} = 24,750$$

Opgave 10:
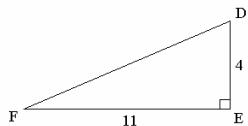
a. $\cos 23° = \dfrac{AB}{7}$

   $AB = 7\cos 23° = 6,44$

b. $\tan \angle D = \dfrac{11}{4}$

 $\angle D = 70°$

Opgave 11:

$$\sin 45° = \dfrac{CD}{6}$$
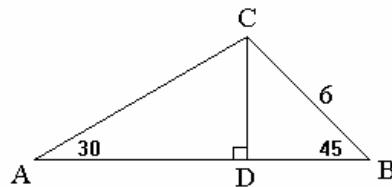
$$CD = 6 \cdot \sin 45° = 6 \cdot \tfrac{1}{2}\sqrt{2} = 3\sqrt{2}$$

$$BD = 3\sqrt{2}$$

$$\tan 30° = \dfrac{CD}{AD} = \dfrac{3\sqrt{2}}{AD}$$

$$AD = \dfrac{3\sqrt{2}}{\tan 30°} = \dfrac{3\sqrt{2}}{\tfrac{1}{3}\sqrt{3}} = \dfrac{3\sqrt{2}}{\tfrac{1}{3}\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{6}}{1} = 3\sqrt{6}$$

$$Opp(\Delta ABC) = \tfrac{1}{2} \cdot (3\sqrt{6} + 3\sqrt{2}) \cdot 3\sqrt{2} = 4\tfrac{1}{2}\sqrt{12} + 9 = 4\tfrac{1}{2} \cdot 2\sqrt{3} + 9 = 9\sqrt{3} + 9$$

Opgave 12:

a. $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b}$

   $\dfrac{\sin 35°}{3} = \dfrac{\sin \beta}{5}$

 $\sin \beta = \dfrac{5 \cdot \sin 35°}{3} = 0,956$
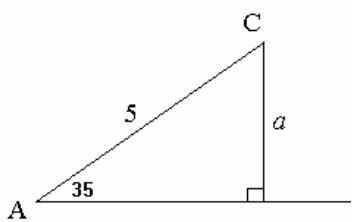
   $\beta = 73° \quad \vee \quad \beta = 107°$

   $\gamma = 180° - 35° - 73° = 72° \quad \vee \quad \gamma = 180° - 35° - 107° = 38°$


b. $\sin 35° = \dfrac{a}{5}$

   $a = 5 \cdot \sin 35° = 2,87$

 $a = 2,87 \quad \vee \quad a \geq 5$

Opgave 13:

a. $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos \angle B$

   $5^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \cdot \cos \angle B$

   $25 = 16 + 36 - 48 \cos \angle B$

   $48 \cos \angle B = 27$
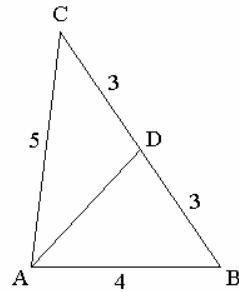
   $\cos \angle B = \frac{27}{48}$

   $\angle B = 56°$

b. $AD^2 = AB^2 + BD^2 - 2 \cdot AB \cdot BD \cdot \cos \angle B$

   $AD^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \frac{27}{48} = 11{,}5$

   $AD = 3{,}39$

Opgave 14:

a. $AM = \sqrt{AD^2 + DM^2} = \sqrt{a^2 + (1\frac{1}{2}a)^2} = \sqrt{a^2 + 2\frac{1}{4}a^2} = \sqrt{3\frac{1}{4}a^2} = \sqrt{\frac{13}{4}a^2} = \frac{1}{2}a\sqrt{13}$

b. $AM = BM = \frac{1}{2}a\sqrt{13}$

   $FM = \sqrt{BM^2 + BF^2} = \sqrt{(\frac{1}{2}a\sqrt{13})^2 + (2a)^2} = \sqrt{3\frac{1}{4}a^2 + 4a^2} = \sqrt{7\frac{1}{4}a^2} = \sqrt{\frac{29}{4}a^2} = $

   $\frac{1}{2}a\sqrt{29}$

c. $FH = \sqrt{EF^2 + EH^2} = \sqrt{(3a)^2 + a^2} = \sqrt{9a^2 + a^2} = \sqrt{10a^2} = a\sqrt{10}$

   $FS = \frac{1}{2}FH = \frac{1}{2}a\sqrt{10}$

   $BS = \sqrt{BF^2 + FS^2} = \sqrt{(2a)^2 + (\frac{1}{2}a\sqrt{10})^2} = \sqrt{4a^2 + 2\frac{1}{2}a^2} = \sqrt{6\frac{1}{2}a^2} = \sqrt{\frac{26}{4}a^2} = \frac{1}{2}a\sqrt{26}$

Opgave 15:

$\sin 60° = \dfrac{CE}{6}$

$CE = 6 \cdot \sin 60° = 6 \cdot \frac{1}{2}\sqrt{3} = 3\sqrt{3}$

$\cos 60° = \dfrac{BE}{6}$

$BE = 6 \cdot \cos 60° = 6 \cdot \frac{1}{2} = 3$

$DF = CE = 3\sqrt{3}$

$\tan 30° = \dfrac{DF}{AF} = \dfrac{3\sqrt{3}}{AF}$

$AF = \dfrac{3\sqrt{3}}{\tan 30°} = \dfrac{3\sqrt{3}}{\frac{1}{3}\sqrt{3}} = 9$
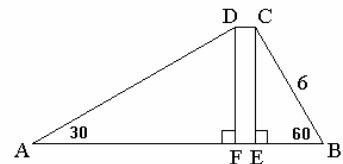
$Opp(ABCD) = \frac{1}{2} \cdot CE \cdot (AB + CD) = \frac{1}{2} \cdot 3\sqrt{3} \cdot (9 + EF + 3 + CD) = 1\frac{1}{2}\sqrt{3} \cdot (12 + 2 \cdot CD) = 36$

$12 + 2 \cdot CD = \dfrac{36}{1\frac{1}{2}\sqrt{3}} = \dfrac{24}{\sqrt{3}} = \dfrac{24}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{24\sqrt{3}}{3} = 8\sqrt{3}$

$2 \cdot CD = 8\sqrt{3} - 12$

$CD = 4\sqrt{3} - 6$

$AB = AF + EF + BE = 9 + 4\sqrt{3} - 6 + 3 = 6 + 4\sqrt{3}$

Opgave 16:

$CP = \sqrt{AC^2 - AD^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24$

$\Delta AKN \infty \Delta APC$ (snavelfiguur)

$\dfrac{AK}{AP} = \dfrac{KN}{PC}$

$\dfrac{x}{7} = \dfrac{KN}{24}$

$KN = \dfrac{24 \cdot x}{7} = \tfrac{24}{7}x$
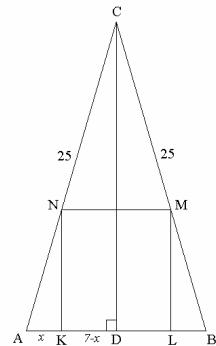
$Opp(KLMN) = KL \cdot KN = 2 \cdot (7 - x) \cdot \tfrac{24}{7}x = 48x - \tfrac{48}{7}x^2$

$Opp' = 48 - \tfrac{96}{7}x$

$-\tfrac{96}{7}x = -48$

$x = 3\tfrac{1}{2}$

$Opp(KLMN) = 48 \cdot 3\tfrac{1}{2} - \tfrac{48}{7} \cdot (3\tfrac{1}{2})^2 = 84$

Gemengde opgave hoofdstuk 4 Algebra en meetkunde.

Opgave 31:

a. $(2a + \sqrt{3})^2 = 4a^2 + 4a\sqrt{3} + 3$

b. $(a + 2\sqrt{3})(a - 2\sqrt{3}) = a^2 - 12$

c. $(2\sqrt{2} + 3\sqrt{8})^2 = (2\sqrt{2} + 3 \cdot 2\sqrt{2})^2 = (2\sqrt{2} + 6\sqrt{2})^2 = (8\sqrt{2})^2 = 128$

d. $\sqrt{\frac{1}{2}} + 6\sqrt{32} = \frac{1}{2}\sqrt{2} + 6 \cdot 4\sqrt{2} = \frac{1}{2}\sqrt{2} + 24\sqrt{2} = 24\frac{1}{2}\sqrt{2}$

e. $\dfrac{\sqrt{2} + 1}{\sqrt{2} - 1} = \dfrac{\sqrt{2} + 1}{\sqrt{2} - 1} \cdot \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1} = \dfrac{2 + 2\sqrt{2} + 1}{2 - 1} = \dfrac{3 + 2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

f. $\dfrac{5}{\sqrt{3} + 1} = \dfrac{5}{\sqrt{3} + 1} \cdot \dfrac{\sqrt{3} - 1}{\sqrt{3} - 1} = \dfrac{5\sqrt{3} - 5}{3 - 1} = \dfrac{5\sqrt{3} - 5}{2} = 2\frac{1}{2}\sqrt{3} - 2\frac{1}{2}$

g. $\dfrac{\sqrt{8} + \sqrt{12}}{2\sqrt{3}} = \dfrac{2\sqrt{2} + 2\sqrt{3}}{2\sqrt{3}} = \dfrac{2\sqrt{2} + 2\sqrt{3}}{2\sqrt{3}} = \dfrac{2\sqrt{2} + 2\sqrt{3}}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{6} + 6}{6} = \frac{1}{3}\sqrt{6} + 1$

h. $(2a + 3\sqrt{2})(2a + 2\sqrt{3}) = 4a^2 + 4a\sqrt{3} + 6a\sqrt{2} + 6\sqrt{6}$

Opgave 32:

a. $3x + \dfrac{6}{2x - 1} = \dfrac{3x(2x - 1) + 6}{2x - 1} = \dfrac{6x^2 - 3x + 6}{2x - 1}$

b. $\dfrac{2x - 1}{x + 2} - \dfrac{x + 2}{x - 4} = \dfrac{(2x - 1)(x - 4) - (x + 2)(x + 2)}{(x + 2)(x - 4)} = \dfrac{2x^2 - 9x + 4 - (x^2 + 4x + 4)}{(x + 2)(x - 4)} = $

$ = \dfrac{2x^2 - 9x + 4 - x^2 - 4x - 4}{(x + 2)(x - 4)} = \dfrac{x^2 - 13x}{(x + 2)(x - 4)}$

c. $\dfrac{a^2}{2a + 5} + \dfrac{a^4}{a - 3} = \dfrac{a^2(a - 3) + a^4(2a + 5)}{(2a + 5)(a - 3)} = \dfrac{a^3 - 3a^2 + 2a^5 + 5a^4}{(2a + 5)(a - 3)}$

d. $\dfrac{3x^2 + 6x}{x^2 + 8x + 12} = \dfrac{3x(x + 2)}{(x + 2)(x + 6)} = \dfrac{3x}{x + 6}$

e. $\dfrac{x^4 - 9x^2 + 8}{x^4 - 1} = \dfrac{(x^2 - 1)(x^2 - 8)}{(x^2 - 1)(x^2 + 1)} = \dfrac{x^2 - 8}{x^2 + 1}$

f. $\dfrac{a^6 - 5a^3 + 4}{6a^3 - 24} = \dfrac{(a^3 - 1)(a^3 - 4)}{6(a^3 - 4)} = \dfrac{a^3 - 1}{6} = \frac{1}{6}a^3 - \frac{1}{6}$

Opgave 33:

a. $\dfrac{1}{x + 1} + \dfrac{3}{2x + 1} = \dfrac{8}{15}$

$\dfrac{2x + 1 + 3(x + 1)}{(x + 1)(2x + 1)} = \dfrac{8}{15}$

$\dfrac{2x + 1 + 3x + 3}{2x^2 + 3x + 1} = \dfrac{8}{15}$

$\dfrac{5x + 4}{2x^2 + 3x + 1} = \dfrac{8}{15}$

$8(2x^2 + 3x + 1) = 15(5x + 4)$

$$16x^2 + 24x + 8 = 75x + 60$$

$$16x^2 - 51x - 52 = 0$$

$$x = \frac{51 \pm \sqrt{2601 + 3328}}{32} = \frac{51 \pm \sqrt{5929}}{32} = \frac{51 \pm 77}{32}$$

$$x = \frac{51 + 77}{32} = 4 \quad \vee \quad x = \frac{51 - 77}{32} = -\frac{13}{16}$$

b. $$\frac{x^2 - 4}{x^2 + 4x + 4} = 2x$$

$$\frac{(x-2)(x+2)}{(x+2)(x+2)} = 2x$$

$$\frac{x-2}{x+2} = 2x$$

$$2x(x+2) = x - 2$$

$$2x^2 + 4x = x - 2$$

$$2x^2 + 3x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{9-16}}{4} = k.n. \text{ dus geen oplossingen}$$

Opgave 34:

a. $x^4 \cdot \sqrt[3]{x} = x^4 \cdot x^{\frac{1}{3}} = x^{4\frac{1}{3}}$

b. $\dfrac{x^{-2}}{x^3} = x^{-5}$

c. $x \cdot \sqrt{\dfrac{1}{x^5}} = x \cdot \sqrt{x^{-5}} = x \cdot (x^{-5})^{\frac{1}{2}} = x \cdot x^{-2\frac{1}{2}} = x^{-1\frac{1}{2}}$

d. $\dfrac{1}{x} \cdot (\sqrt[4]{x^3})^8 = x^{-1} \cdot (x^{\frac{3}{4}})^8 = x^{-1} \cdot x^6 = x^5$

e. $\dfrac{x^3 \cdot x^{-5}}{\sqrt{x}} = \dfrac{x^{-2}}{x^{\frac{1}{2}}} = x^{-2\frac{1}{2}}$

f. $(x\sqrt{x})^{-3} = (x^{1\frac{1}{2}})^{-3} = x^{-4\frac{1}{2}}$

Opgave 35:

a. $F = (2000 - 16{,}3 \cdot 60)(-5 - -20)^{-1{,}668} = 11{,}2$ dus maximaal 11 minuten

b. $(2000 - 16{,}3v)(-5 + 18)^{-1{,}668} = 20$

$(2000 - 16{,}3v) \cdot 13^{-1{,}668} = 20$

$2000 - 16{,}3v = \dfrac{20}{13^{-1{,}668}} = 1442{,}4$

$-16{,}3v = -557{,}6$

$v = 34{,}2$ dus $34 \, ^{km}\!/_{uur}$

c. $tijd = \frac{10}{40} = 0{,}25$ uur dus $F = 15$ en $v = 0$

$2000 \cdot (-5 - T)^{-1{,}668} = 15$

$y_1 = 2000 \cdot (-5 - X)^{-1{,}668}$ en $y_2 = 15$

Intersect geeft $x = -23{,}8$ dus $T < -23{,}8$

Opgave 36:

a. $A = 136$

$a \cdot 40^{0,67} = 136$

$a = \dfrac{136}{40^{0,67}} = 11,5$

b. $A = 11,5 \cdot 275^{0,67} = 496\,\text{dm}^2$

c. $A = 1,16$

$11,5 m^{0,67} = 1,16$

$m^{0,67} = 0,1$

$m = \sqrt[0,67]{0,1} = 0,033\,\text{kg} = 33\,\text{g}$

Opgave 37:

$\cos 30° = \dfrac{AK}{5}$
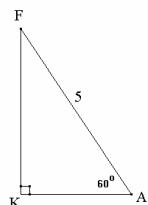
$AK = 5\cos 30° = 5 \cdot \tfrac{1}{2}\sqrt{2} = 2\tfrac{1}{2}\sqrt{2}$

$FK = 5\sin 30° = 5 \cdot \tfrac{1}{2} = 2\tfrac{1}{2}$

$KL = 2 \cdot 2\tfrac{1}{2} + 5 = 10$

$KN = 2 \cdot 2\tfrac{1}{2}\sqrt{3} = 5\sqrt{3}$

$Opp = 10 \cdot 5\sqrt{3} = 50\sqrt{3}$

Opgave 38:

$h = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$

$Opp\Delta = \tfrac{1}{2} \cdot 4 \cdot 2\sqrt{3} = 4\sqrt{3}$

$Opp(\text{zeshoek}) = 6 \cdot 4\sqrt{3} = 24\sqrt{3}$
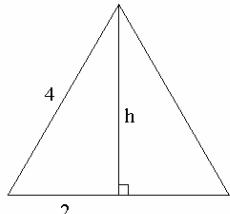
$Opp(\text{ingeschreven cirkel}) = \pi \cdot (2\sqrt{3})^2 = 12\pi$

$Opp(\text{omgeschreven cirkel}) = \pi \cdot 4^2 = 16\pi$

$Opp(\text{zeshoek—ingeschreven cirkel}) = 24\sqrt{3} - 12\pi$

$Opp(\text{omgeschreven zeshoek—zeshoek}) = 16\pi - 24\sqrt{3}$

$Opp = \tfrac{1}{2}(16\pi - 24\sqrt{3}) + 24\sqrt{3} - 12\pi = 8\pi - 12\sqrt{3} + 24\sqrt{3} - 12\pi = 12\sqrt{3} - 4\pi$

Opgave 39:

$Opp(ABCD) = 25$

$Opp\Delta DRS = \tfrac{1}{2} \cdot (25 - 15) = 5$

$DS = 5 - x$

$DR = \tfrac{1}{2}(5 - x)$

$Opp\Delta DRS = \tfrac{1}{2} \cdot (5-x) \cdot \tfrac{1}{2}(5-x) = \tfrac{1}{4}(5-x)^2 = 5$

$(5 - x)^2 = 20$

$5 - x = \sqrt{20} \quad \vee \quad 5 - x = -\sqrt{20}$

$x = 5 - \sqrt{20} \quad \vee \quad x = 5 + \sqrt{20} \text{ (vervalt)}$

Opgave 40:

a. $\tan 60° = \dfrac{h}{3}$

$h = 3 \cdot \tan 60° = 3 \cdot \sqrt{3} = 3\sqrt{3}$

$AC = \sqrt{10^2 + (3\sqrt{3})^2} = \sqrt{100 + 27} = \sqrt{127}$

b. $\cos 60° = \dfrac{3}{AD}$
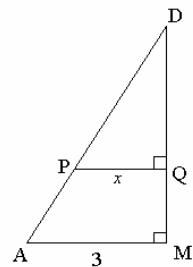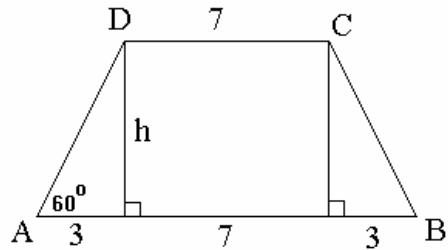
$AD = \dfrac{3}{\cos 60°} = \dfrac{3}{\frac{1}{2}} = 6$

$omtrek = 6 + 7 + 6 + 13 = 32$

$Opp = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2} \cdot 3\sqrt{3} \cdot (13 + 7) = 30\sqrt{3}$

c. $\dfrac{DQ}{3\sqrt{3}} = \dfrac{x}{3}$

$DQ = \dfrac{3\sqrt{3} \cdot x}{3} = x\sqrt{3}$

$OppPRCD = \frac{1}{2} \cdot x\sqrt{3} \cdot (2x + 7 + 7) = \frac{1}{2}x\sqrt{3} \cdot (2x + 14) = 15\sqrt{3}$

$y_1 = \frac{1}{2}x\sqrt{3} \cdot (2x + 14)$ en $y_2 = 15\sqrt{3}$

Intersect geeft: $x = 1,72$

$QM = 3\sqrt{3} - DQ = 3\sqrt{3} - 1,72\sqrt{3} = 2,22$

Opgave 41:

a. $MT = \sqrt{a^2 + (3a)^2} = \sqrt{a^2 + 9a^2} = \sqrt{10a^2} = a\sqrt{10}$

b. $CM = \sqrt{(2a)^2 + (1\frac{1}{2}a)^2} = \sqrt{4a^2 + 2\frac{1}{4}a^2} = \sqrt{6\frac{1}{4}a^2} = 2\frac{1}{2}a$

c. neem P het midden van SC

$AC = \sqrt{(3a)^2 + (2a)^2} = \sqrt{9a^2 + 4a^2} = \sqrt{13a^2} = a\sqrt{13}$

$AP = \frac{3}{4}AC = \frac{3}{4} \cdot a\sqrt{13} = \frac{3}{4}a\sqrt{13}$

$AN = \sqrt{AP^2 + PN^2} = \sqrt{(\frac{3}{4}a\sqrt{13})^2 + (1\frac{1}{2}a)^2} = \sqrt{29\frac{1}{4}a^2 + 2\frac{1}{4}a^2} = \sqrt{31\frac{1}{2}a^2} = a\sqrt{31\frac{1}{2}}$

d. van M naar N is $\frac{3}{4}a$ naar rechts, $1\frac{1}{2}a$ naar achter en $1\frac{1}{2}a$ omhoog

$MN = \sqrt{(\frac{3}{4}a)^2 + (1\frac{1}{2}a)^2 + (1\frac{1}{2}a)^2} = \sqrt{\frac{9}{16}a^2 + 2\frac{1}{4}a^2 + 2\frac{1}{4}a^2} = \sqrt{5\frac{1}{16}a^2} = \sqrt{\frac{81}{16}a^2} = \frac{9}{4}a$

Opgave 42:

Stel $BP = x$ dan is $PQ = \frac{3}{4}x$ en $AP = 4 - x$

$Opp(APQR) = \frac{3}{4}x(4 - x) = -\frac{3}{4}x^2 + 3x$

$Opp' = -1\frac{1}{2}x + 3 = 0$

$-1\frac{1}{2}x = -3$

$x = 2$

$Opp = 3$